Accurate Single Shot Phase Measurement Techniques for Optical Metrology

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Optical Metrology … (or wavefront sensing)

- Required for all optical components (flats, spherical, aspherics, and freeform) used in various devices

- Due to availability of advanced fabrication tools, aspheric and freeform optics is finding ever increasing usage … number of elements required for correction is reduced.

- Computational methods for accurate phase estimation are critical to metrology systems

- Common approaches to wavefront sensing:

  - Interferometry (on axis and off axis … CGH based, multi-wavelength, …)
  - Non-interferometric phase measurement
  - Other (contact profilometry, chromatic confocal scanning, …)

- **Single-shot systems**: Estimate phase from one intensity measurement
  - Robust to mechanical vibrations (due to sub-milli-second exposure)
  - Potentially require simpler hardware
  - Suitable for realizing portable systems for in-situ measurements

Plan for the talk

- Single shot interferometric phase measurement with sparse optimization
  - Unified treatment of off-axis and on-axis configurations
  - Resolution advantage
  - Noise advantage

- Transport-of-intensity phase retrieval
  - Sampling advantage of TIE over interferometry
  - Single-shot measurement with a custom prism-mirror module

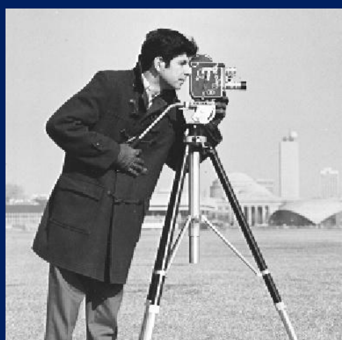
- Fourier phase retrieval
  - Brief discussion on stagnation issues
  - Sparsity and vortex illumination for algorithm stability

Summary and future directions

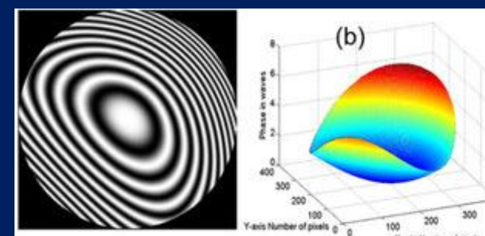
Image sparsity and wavefront sparsity

Natural images are sparse in gradient domain or in suitable bases (like wavelet transform)

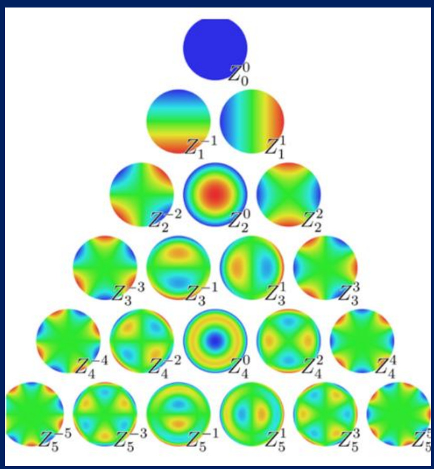
$g(x, y)$ $|\nabla g(x, y)|$



Phase functions of interest to wavefront sensing are sparse in Zernike basis



$$\theta(x, y) = \sum_{n=0}^{N} a_n \, Z_n(x, y)$$



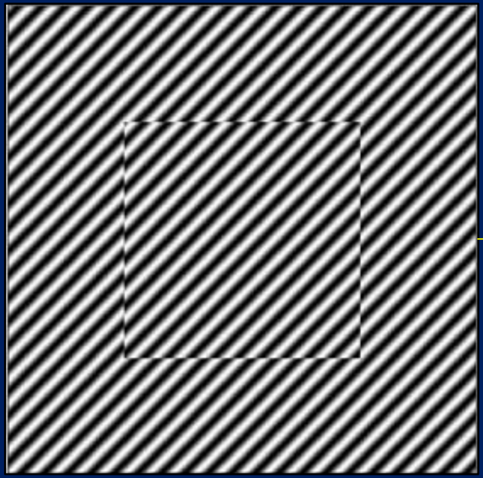
Degrees of freedom in 2D signal = number of significant coefficients

In quantitative imaging or wavefront sensing, the number of unknowns to be recovered is therefore implicitly much less compared to number of pixels.

The main idea is that the solutions we are seeking have a spatial structure or local correlations that may be utilized by appropriate algorithms.

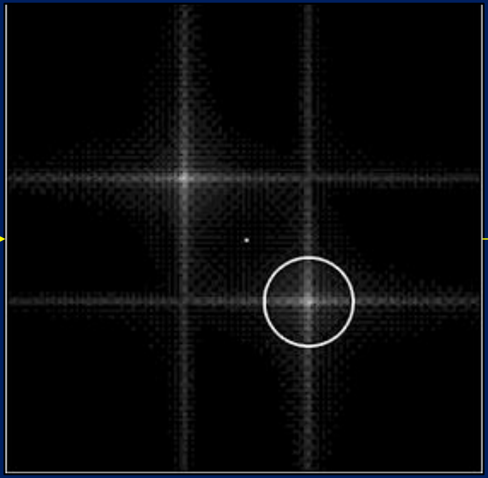
Topic #1: Single-shot interferometry for wavefront sensing

Off-axis interferogram



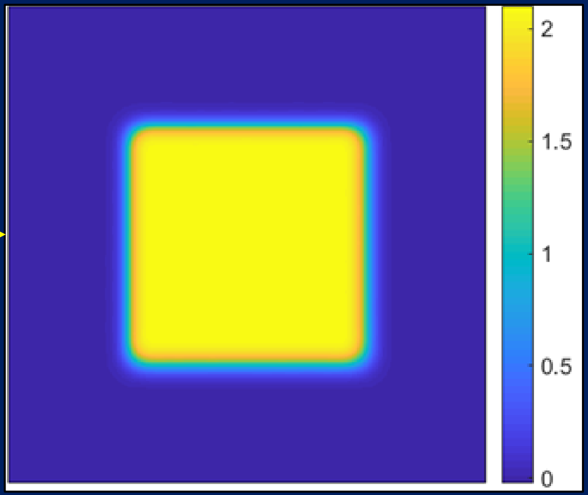
FT

Fourier filtering



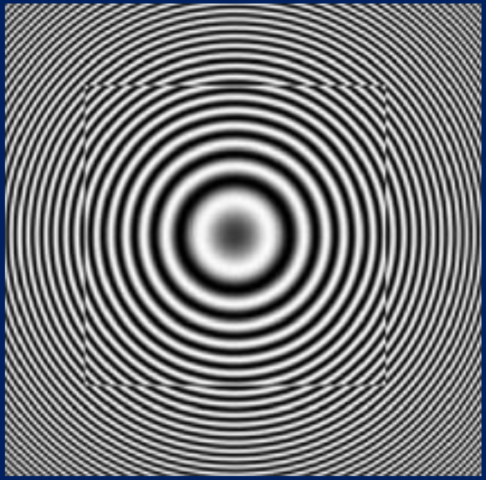
IFT

Phase recovery



Low spatial resolution

On-axis interferogram



Traditionally, no suitable single-shot method available for interferogram demodulation. Phase-shifting is required.

Single-shot accurate phase recovery appears to be difficult, even though all the information is present in the data.

Single-shot phase recovery problem does not have a unique solution

Interference equation

$$|O|^2 + 2|R||O|\cos(\phi_o - \phi_R) + (|R|^2 - H) = 0$$

$$|O| = -|R|\cos(\phi_o - \phi_R) \pm \sqrt{H - |R|^2 \sin^2(\phi_o - \phi_R)}$$

Even when *R* is known *O* has infinite number of solutions !

Complex object-wave recovery by optimization

$$C(O,O^*) = \frac{1}{2}\left\| H - (|O|^2 + |R|^2 + OR^* + O^*R) \right\|^2 + \alpha \psi(O,O^*)$$

L2-norm squared data fit

Physically desirable constraint on the solution

- Constraint can be designed to make pixels in unknown solution talk to each other
- Resolution loss problem in off-axis interferometry can be addressed
- On-axis and off-axis problems can be treated in a unified manner
- Noise advantage beyond single-pixel based limits

Iterative optimization algorithms based on complex or Wirtinger derivatives
[Opt. Express (2013), Applied Opt. (2014), Phys. Rev. A (2015), JOSA A (2017), JOSA A (2019), …]

Readily available deep learning tools allow automatic differentiation for computing functional gradients required to run iterative algorithms.

Sample choices for constraint functions:

$$\psi(g,g^*) = \sum_{\substack{all \\ pixels}} |\nabla g|^2$$

Encourages smooth solutions
(e.g. wavefront from an optical surface)

$$\psi(g,g^*) = \sum_{\substack{all \\ pixels}} |\nabla g|$$

Total Variation or TV (L1-norm of gradient)
Encourages sharp edges and piecewise constant solutions

$$\psi(g,g^*) = \sum_{\substack{all \\ pixels}} \left[\sqrt{1 + \frac{|\nabla g|^2}{\delta^2}} - 1 \right]$$

Modified Huber penalty
Combination of two … suitable for most realistic objects

$$\theta(x,y) = \sum_n a_n \, Z_n(x,y)$$

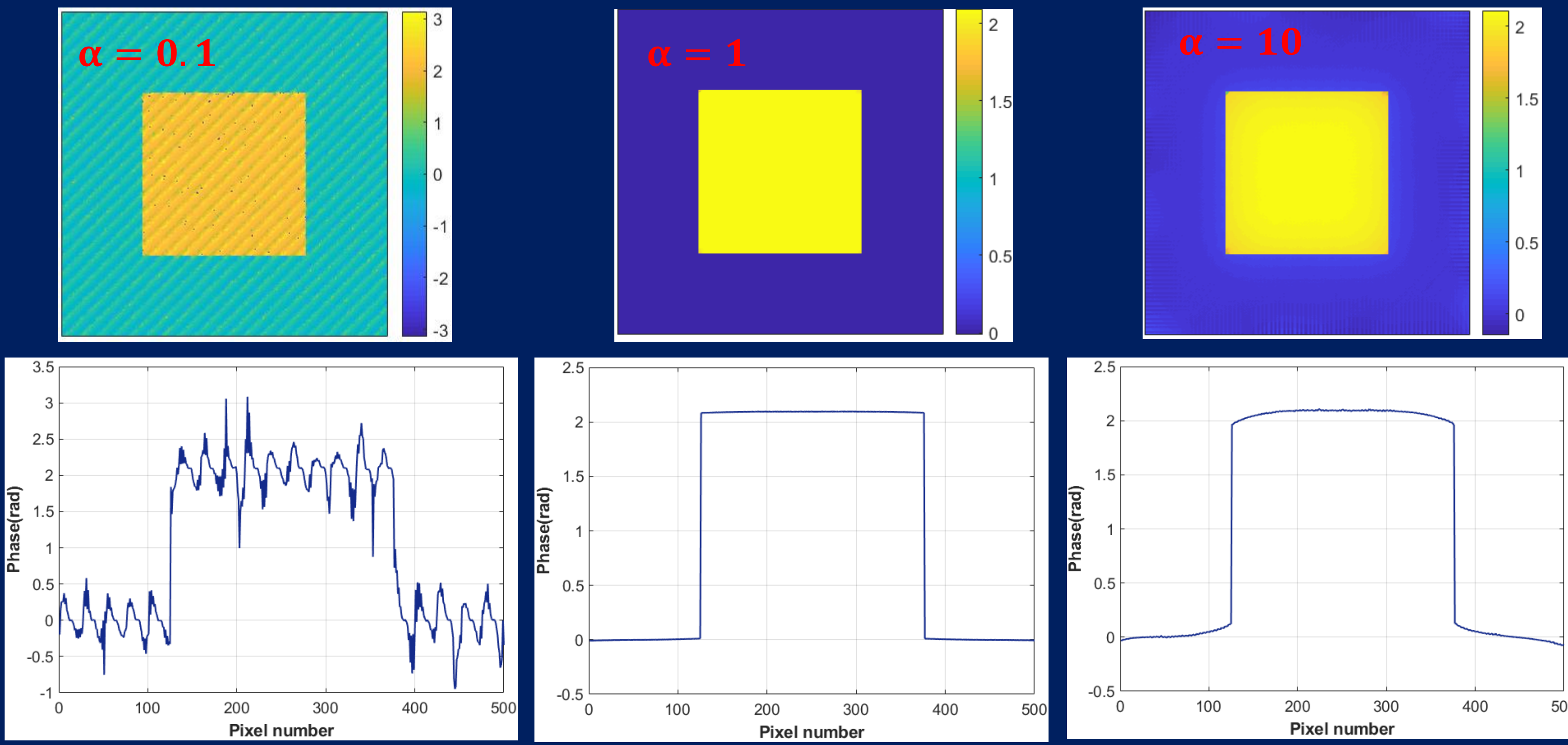
$$\psi(\theta) = \sum_n |a_n|$$

Encourages sparse solution in terms of a small number of Zernike coefficients

Trial of optimization approach …

Minimize $$C(O, O^*) = \left\| H - |O + R|^2 \right\|_2^2 + \alpha \; TV(O, O^*)$$

Recovered phase maps

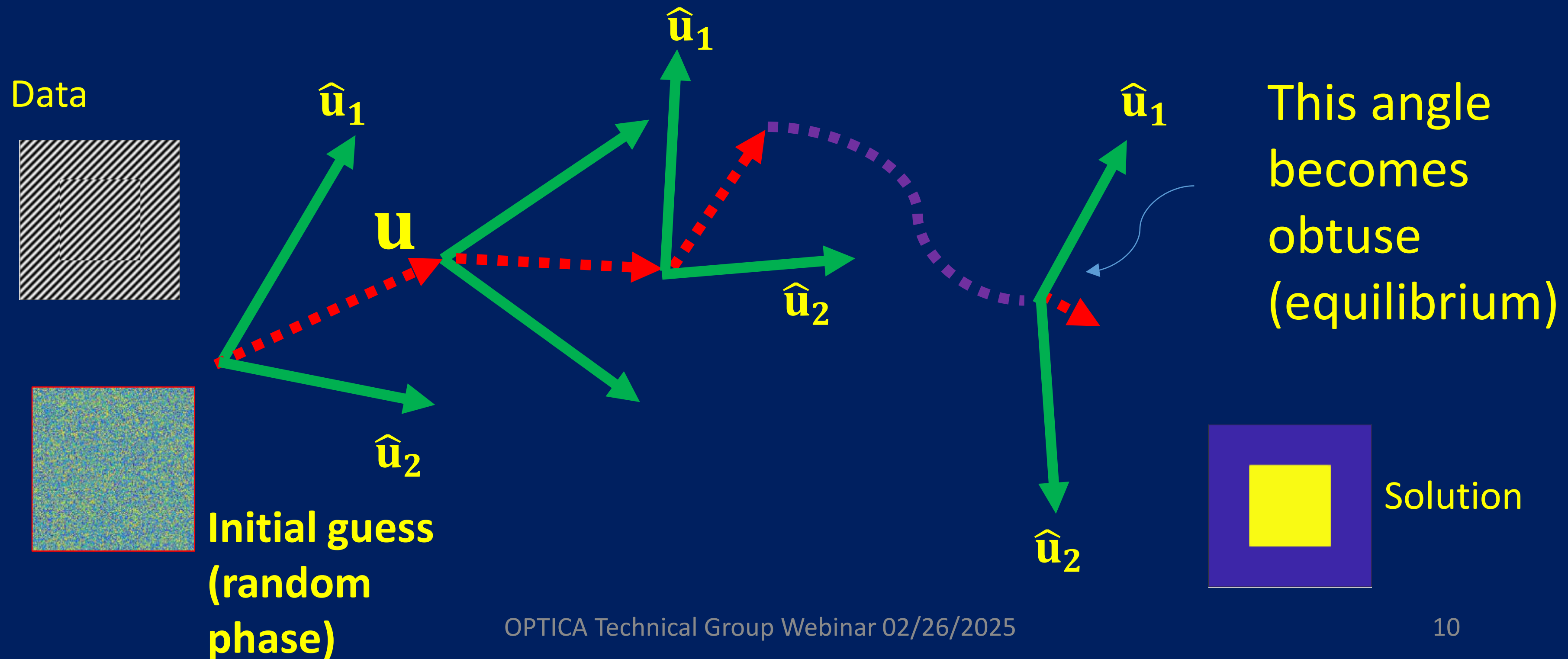


In principle capable of providing full resolution but needs tedious tuning of α .

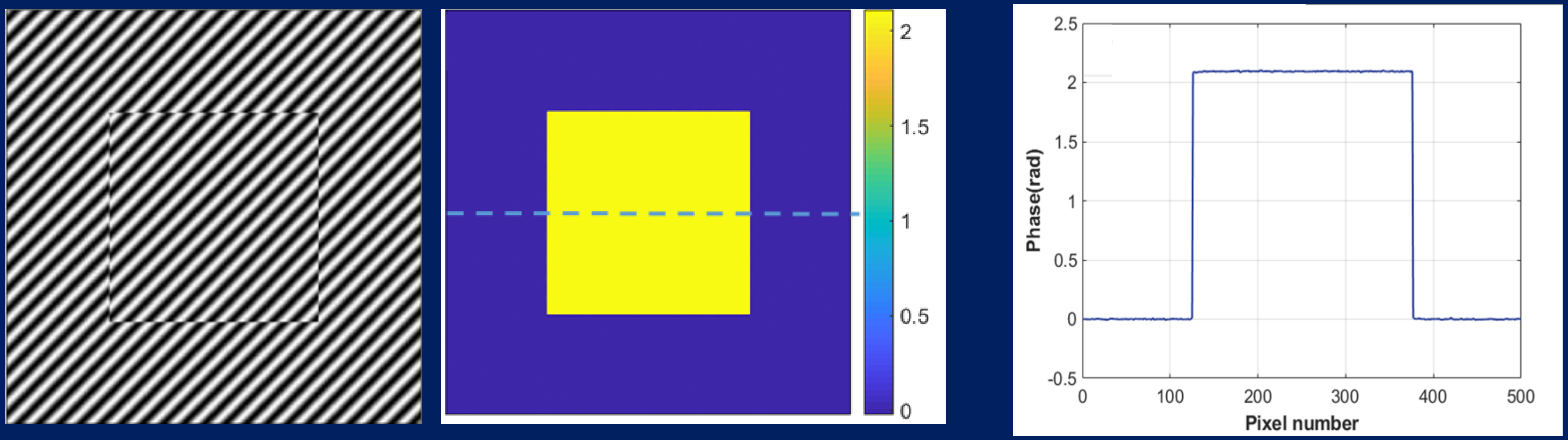
Mean Gradient Descent (MGD): Optimization without any free parameter [JOSA A (2017, 2019), Appl. Optics (2021)]

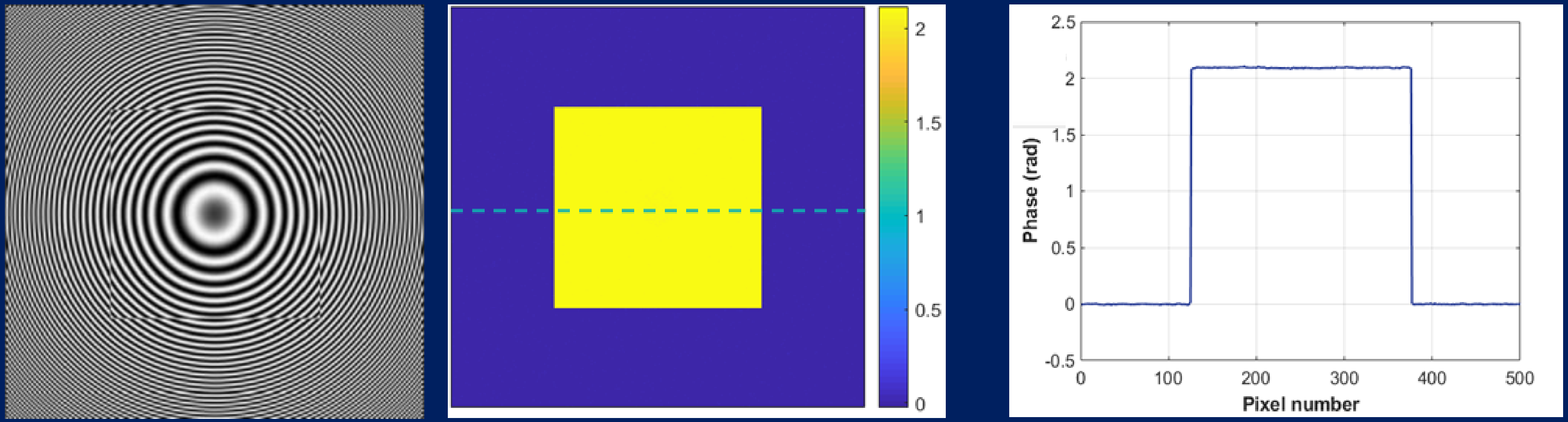
Push the solution in direction that bisects two descent directions …

$$\hat{\mathbf{u}}_1 = \frac{-\nabla_{O^*} C_{err}}{\left\| \nabla_{O^*} C_{err} \right\|} \qquad \hat{\mathbf{u}}_2 = \frac{-\nabla_{O^*} C_{TV}}{\left\| \nabla_{O^*} C_{TV} \right\|} \qquad \boldsymbol{u} = \frac{\hat{\mathbf{u}}_1 + \hat{\mathbf{u}}_2}{2}$$



Full resolution phase recovery from single interferogram with MGD …





RMS phase error $< 1/\sqrt{N}$ (N = avg. photons/pixel)

[JOSA A (2019)]

How accurately can we recover a wavefront phase map ?

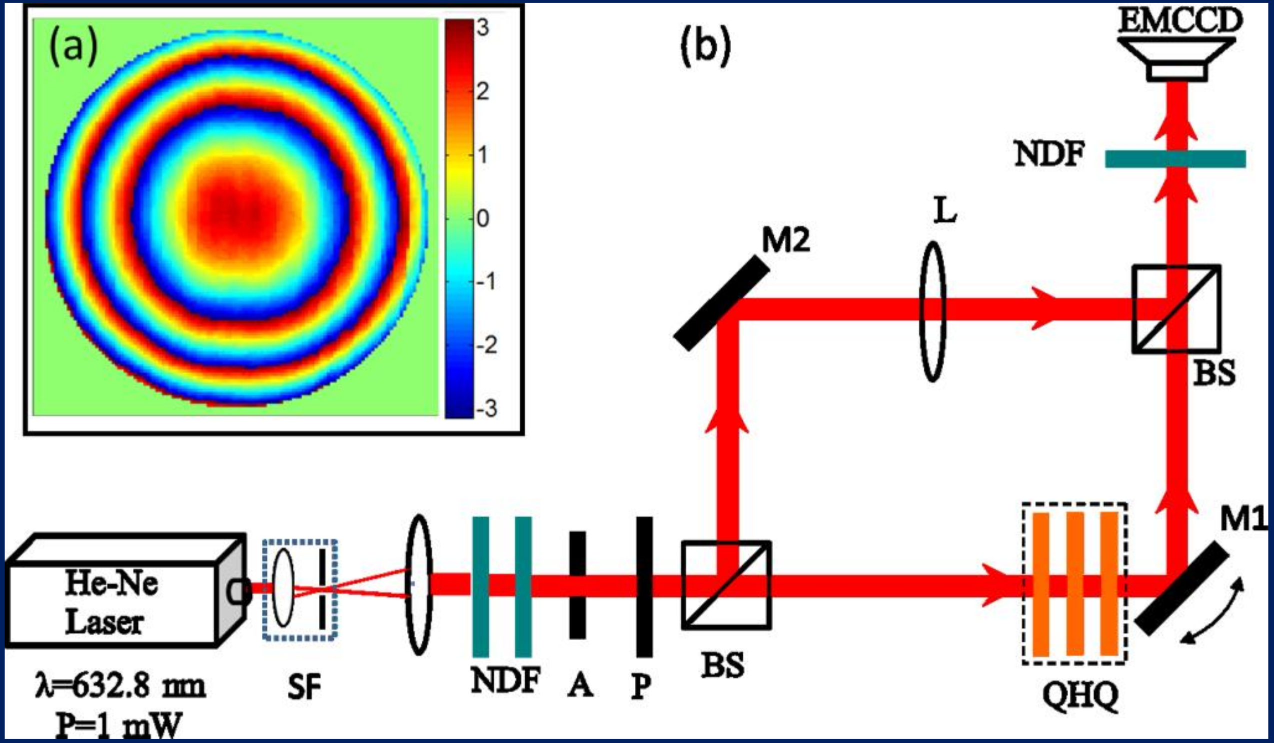
Ultimate noise limit considered traditionally is the shot noise or $\sqrt{N}$ noise …
this limit is however based on single-pixel based analysis.

Accuracy is a function of methodology used for information recovery

Phase shifting solution

$$\phi_o - \phi_R = \arctan\left[\frac{H(3\pi/2) - H(\pi/2)}{H(0) - H(\pi)}\right]$$

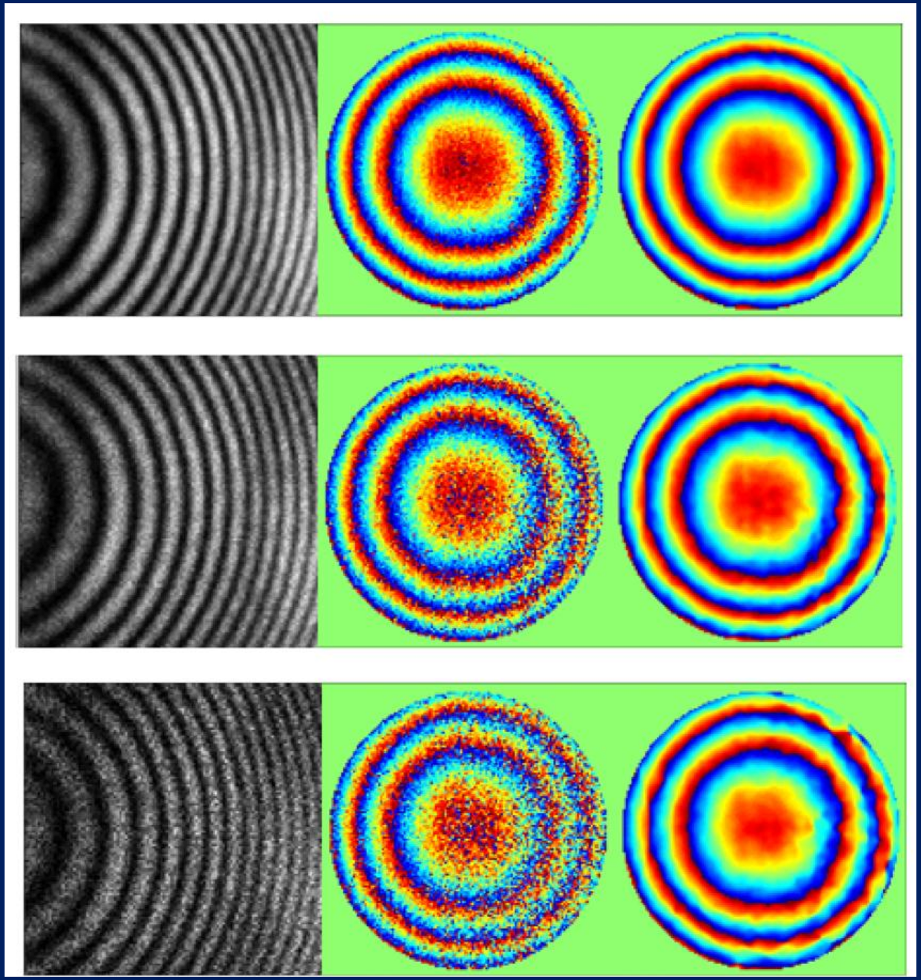
All pixels are treated
Independently in parallel



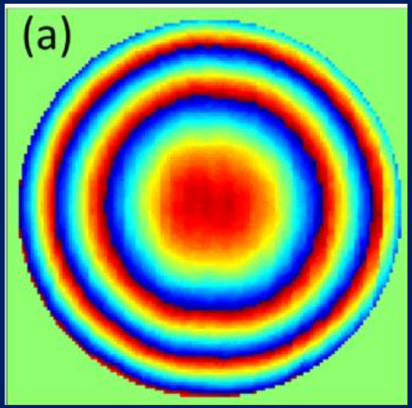
[Physical Review A (2015)]

Phase recovery at low light levels

Interference pattern | Traditional Phase shifting solution | Optimization solution



Ground truth phase map



200 photons / pixel

50 photons / pixel

10 photons / pixel

(> 5x better than single pixel based shot-noise limit)

[Physical Review A (2015)]

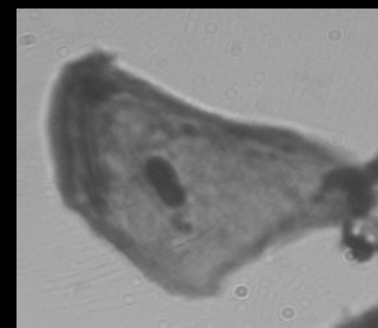
Quantitative Phase Microscopy system:

http://www.holmarc.com/dhm.php

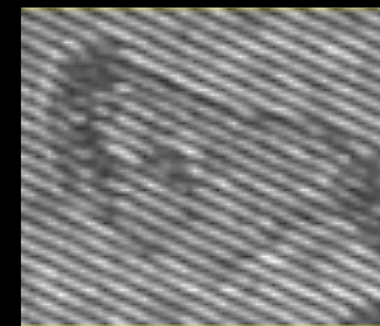
- Single-shot phase reconstruction
- Dual mode … brightfield and phase by switching illumination

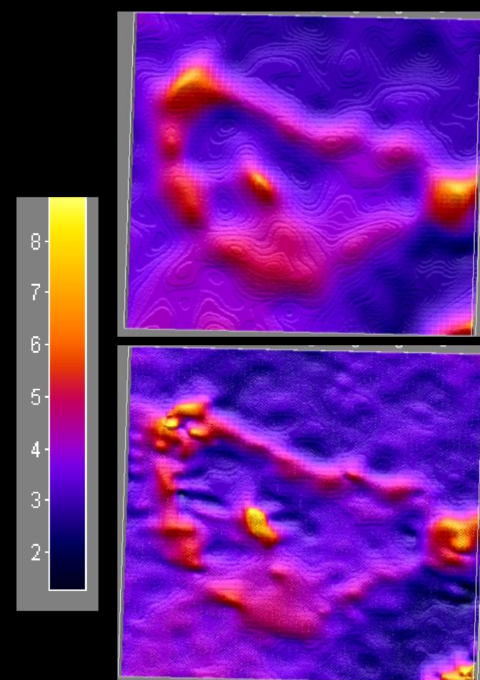
Cervical cell imaging (~ 40 μm cell size) [J. Biophotonics (2019)]

Brightfield



ROI hologram





Phase Image (Fourier Filtering)

Phase Image (Optimization)

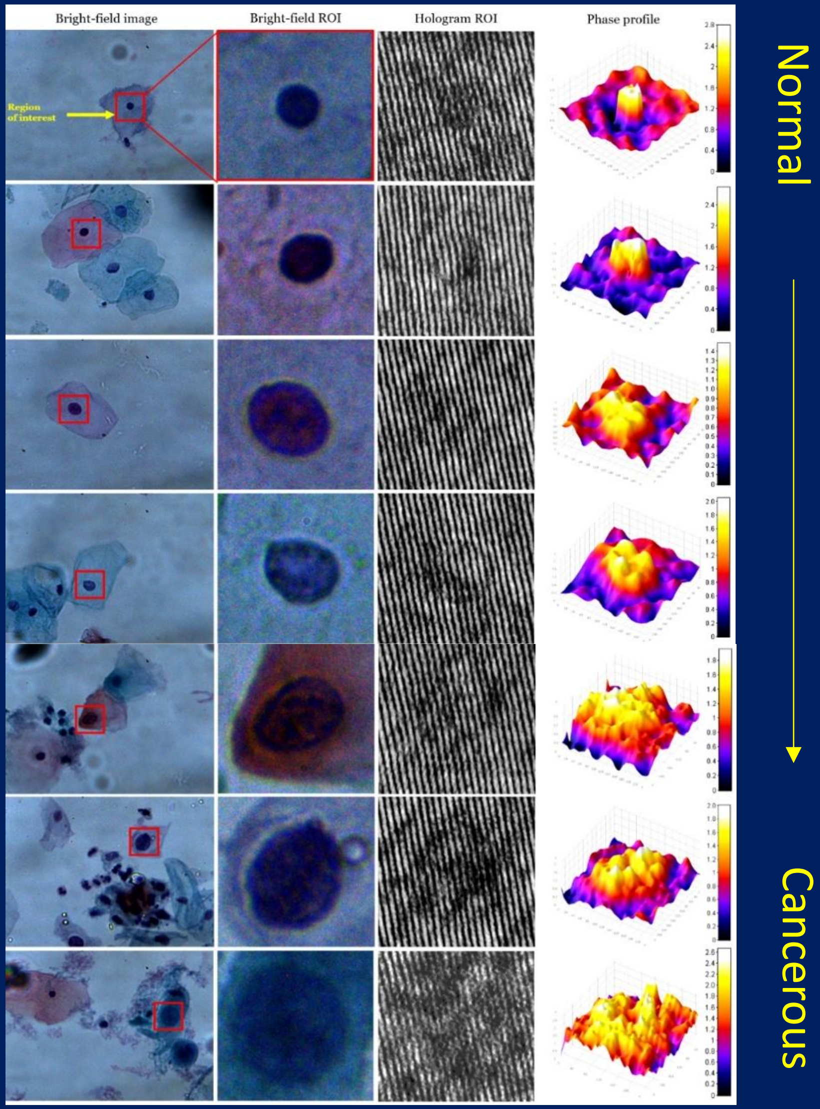
<u>ROI-capability</u>

Quantitative Phase Microscopy system:

http://www.holmarc.com/dhm.php

- Single-shot phase reconstruction
- Dual mode … brightfield and phase by switching illumination

Cervical cell imaging (~ 10 μm cell nucleus) [J. Biophotonics (2019)]



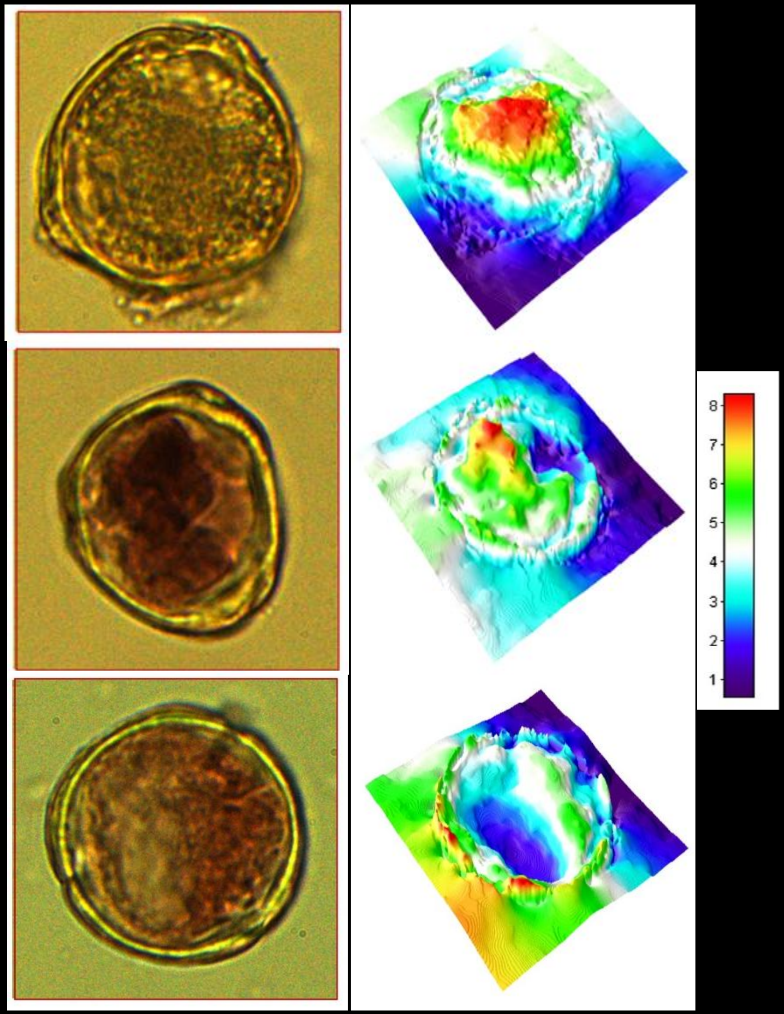
(With All India Institute of Medical Sciences Delhi)

Quantitative Phase Microscopy system:

http://www.holmarc.com/dhm.php

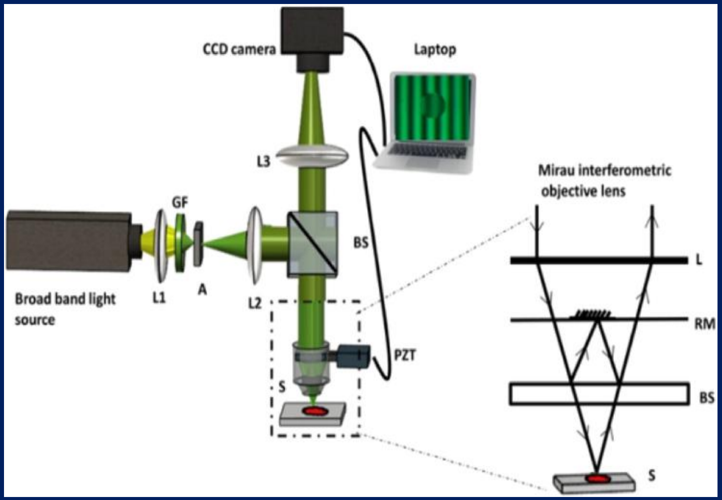
- Single-shot phase reconstruction
- Dual mode … brightfield and phase by switching illumination

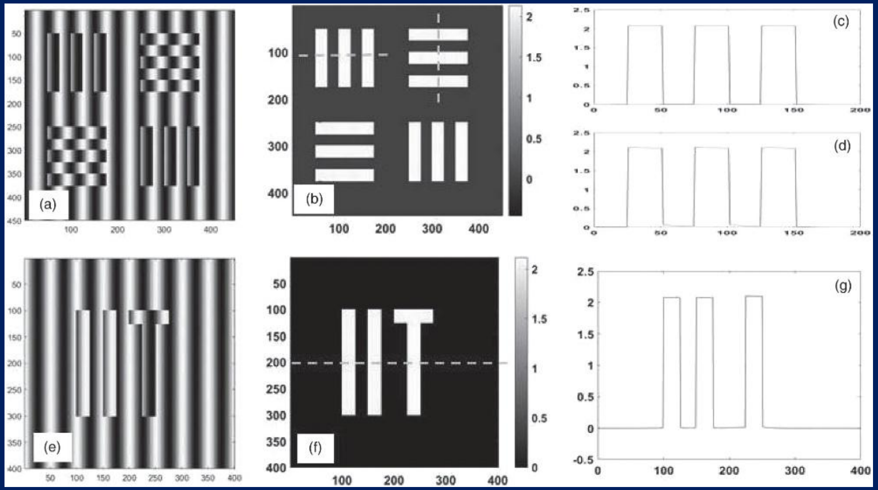
Label-free pollen classification [Quant. Plant. Biol. (2023)]

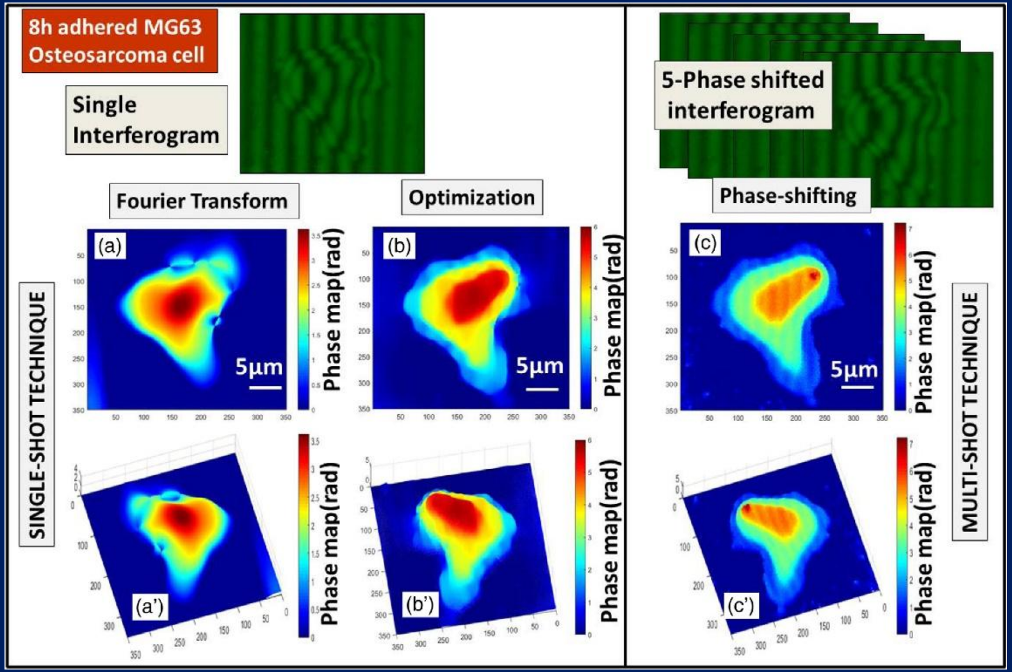


(With National Institute of Plant Genomic Research Delhi)

Low-density fringes on a Mirau LED interferometer [Appl. Opt. (2024)]



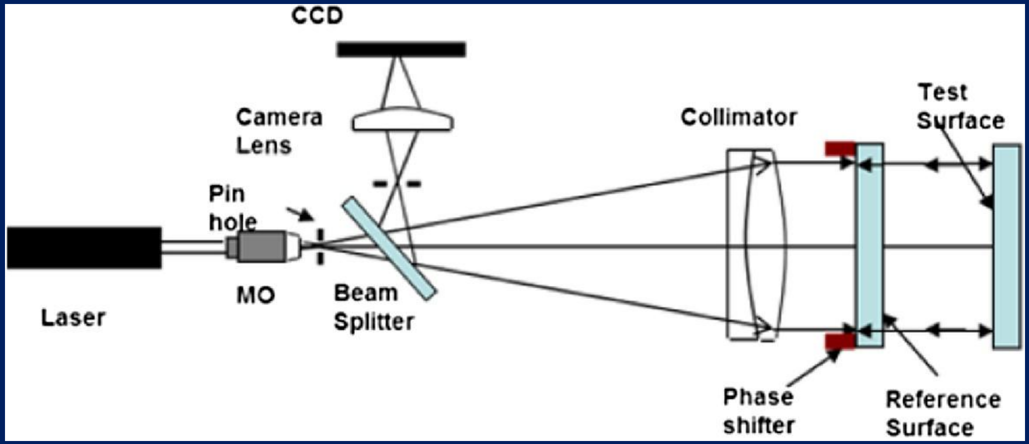


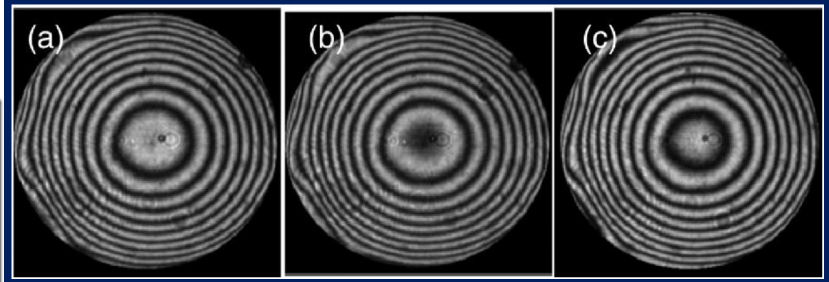


- Calibration of reference beam needed for use in the algorithm

- Phase shifting hardware (piezo etc.) may be bypassed

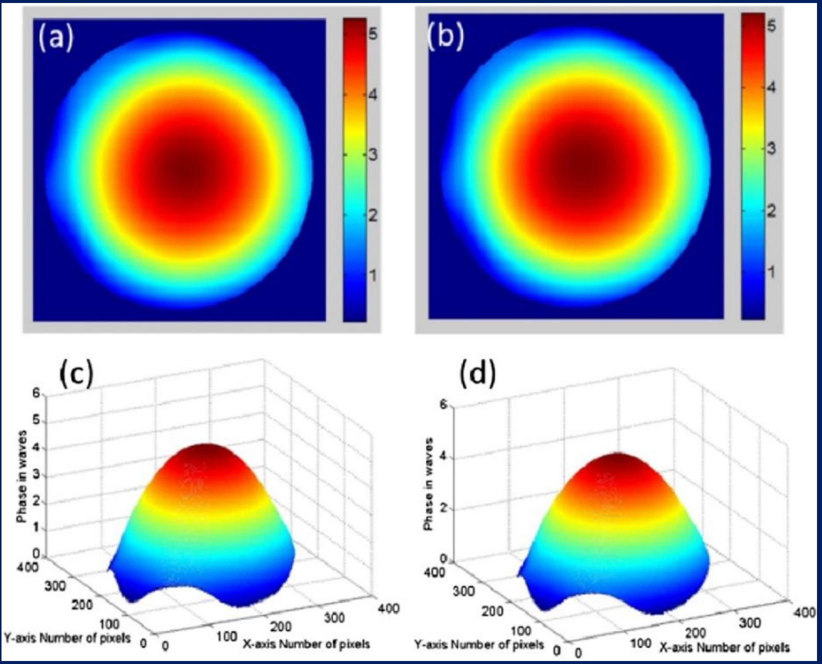
Circular fringes on a Fizeau interferometer [Appl. Opt. (2014)]





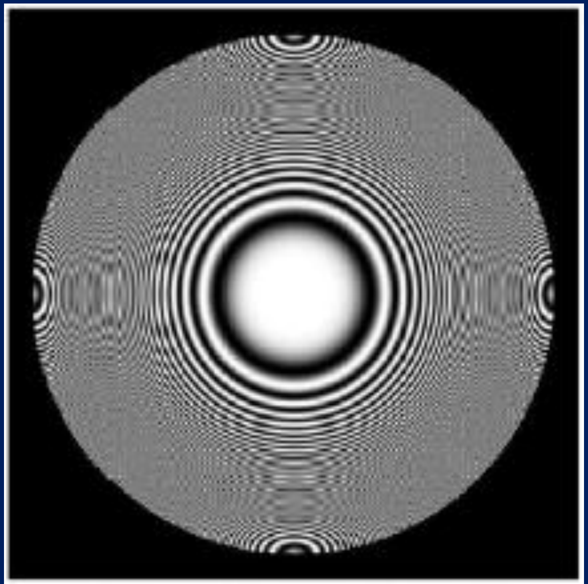
Three (out of nine) phase-shifted frames (from commercial Fizeau system)

Single-shot optimization **Phase-shifting solution**



- Sparsity in Zernike basis is used in optimization
- Two solutions match with < $\frac{\lambda}{40}$ RMS error
- Single-shot method offers multiple advantages
  - Easy to set up the system
  - Stringent vibration isolation is not needed

Topic #2: High fringe density problem in interferometry of aspherics



- As the test and null surface deviate from each other, the fringes get dense and are impossible to sample as per the Nyquist criterion.

- Long standing problem

Curious question: Phase $\theta(x, y)$ of aspheric wavefront is quite smooth. Why does sampling problem arise when measuring such a smooth function?

Surface sag equation

$$Z(s) = \frac{Cs^2}{1 + \sqrt{1 - (1+k)C^2 s^2}} + A_4 s^4 + A_6 s^6 + A_8 s^8 + \ldots$$

Bandwidth of $\theta(x,y)$ vs. $exp[i\,\theta(x,y)]$

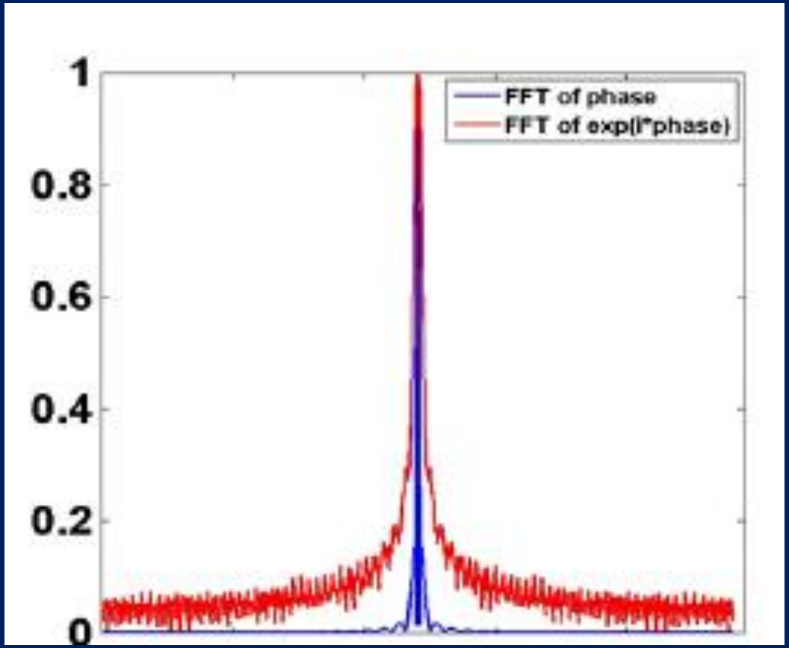
- Interferometry does not measure $\theta(x,y)$ directly.

- The cross-term of an interference pattern contains $\cos[\theta(x,y)]$ which is comparatively a high-bandwidth function.

- This is the mathematical origin of the high fringe density sampling problem.

- *We need a framework where phase is not represented in terms of sines and cosines. Answer: Transport of Intensity Equation*

Normalized FFT magnitude on log-scale



Transport of Intensity Equation (TIE) … [M R Teague, JOSA (1983)]

Helmholtz equation

$$(\nabla^2 + k^2)\, u = 0$$

Paraxial (Fresnel zone) form

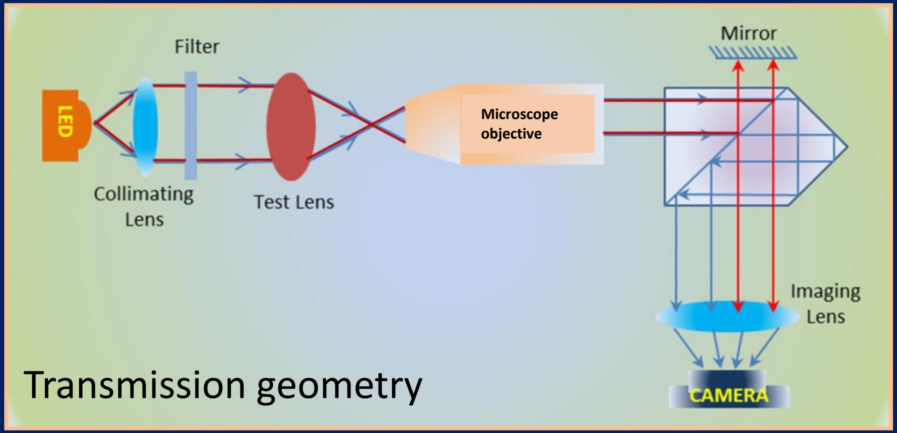
$$\left(i\,\frac{\partial}{\partial z} + \frac{1}{2k}\nabla_{xy}^2 + k \right) u = 0$$

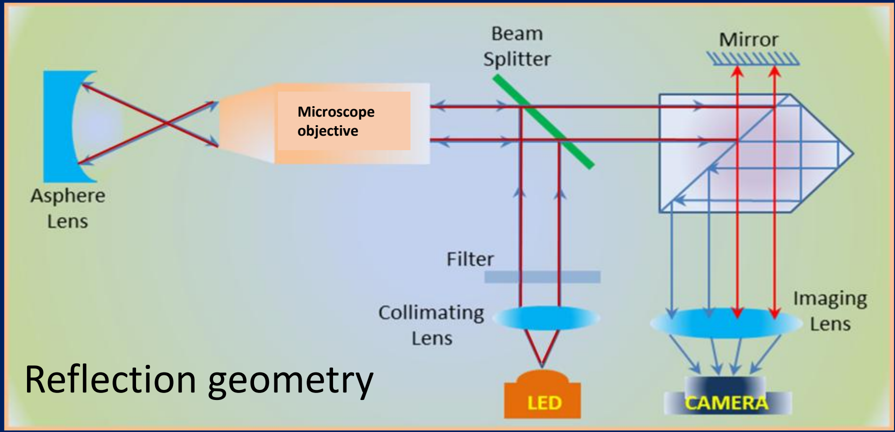
Substituting $u = \sqrt{I}\, exp(i\,\theta)$
leads to a transport equation:

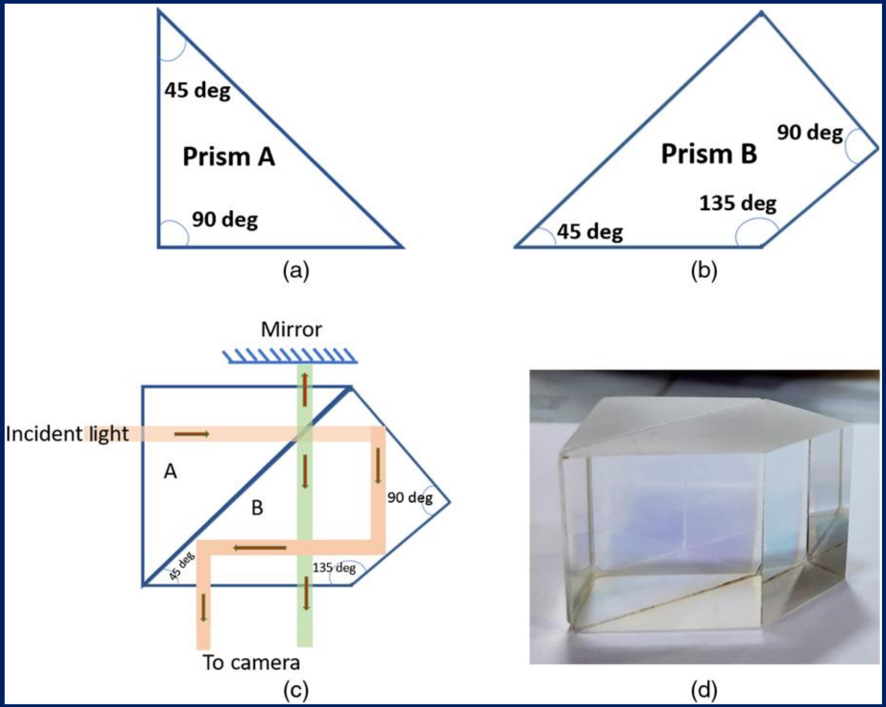
$$-k\,\frac{\partial I}{\partial z} = \nabla_{xy} \cdot (\, I\, \nabla_{xy}\, \theta)$$

- Longitudinal intensity derivative is related to the transverse phase gradient.
- Phase is not represented with arctan(…)
- Amplitude/phase are de-coupled and sampling requirements are highly reduced.
- Solution is automatically in unwrapped form.

Custom-fabricated prism-mirror module to get $\frac{\partial I}{\partial z}$ in one-shot







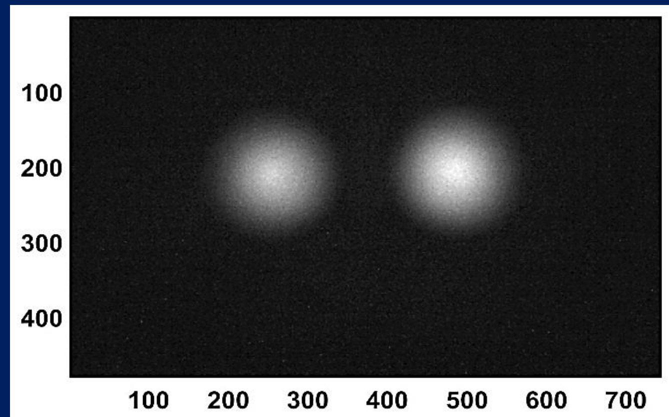
(Fabricated at IRDE, Dehradun)

- The mirror is adjustable and provides required defocus for one wavefront.

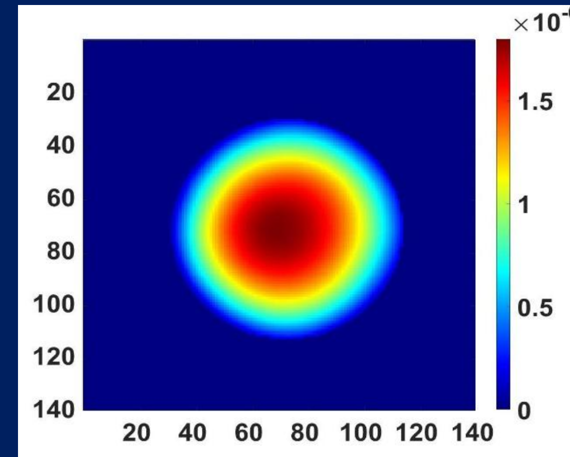
- The *<u>same camera</u>* can now record two copies of the wavefront in two halves of the sensor chip without any sampling problem.

[Opt. Engg. (2022)]

Metrology of 3 mm lens (which can be measured with interferometer)

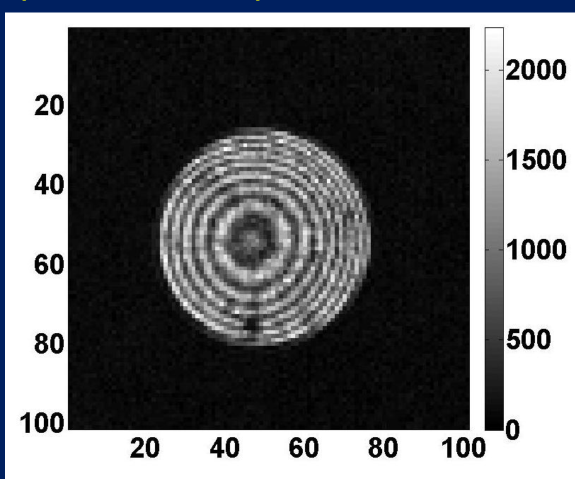


Camera record of two z-displaced wavefronts

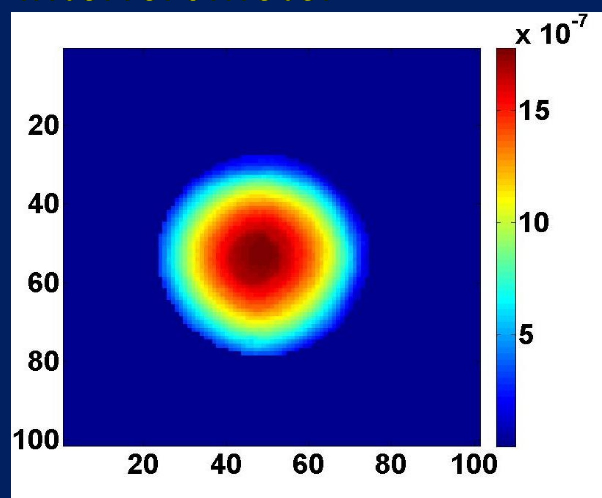


TIE solution

Fizeau Interferometry (one frame)



Phase map from commercial interferometer

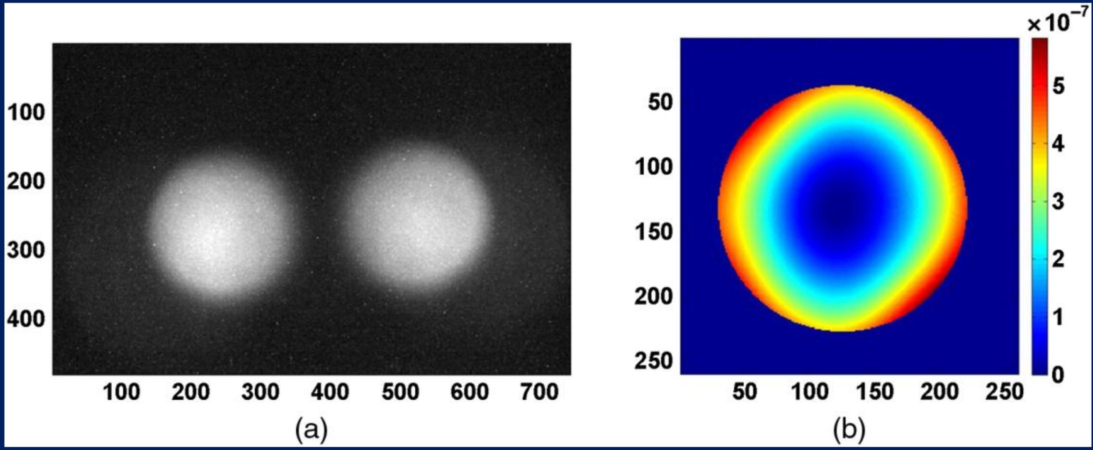


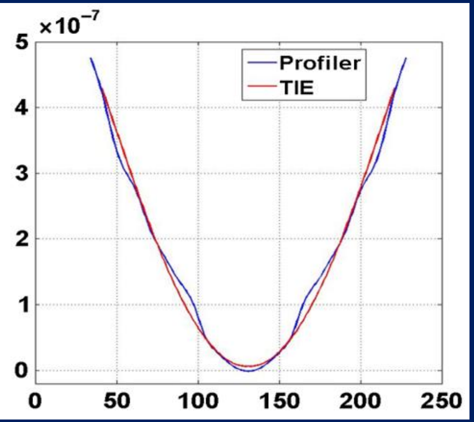
PV error < $\frac{\lambda}{10}$

Metrology of a large aspheric reflecting IR optic (~ 3 cm diameter) (faces high fringe density problem) :

Camera record of two z-displaced wavefronts

TIE solution



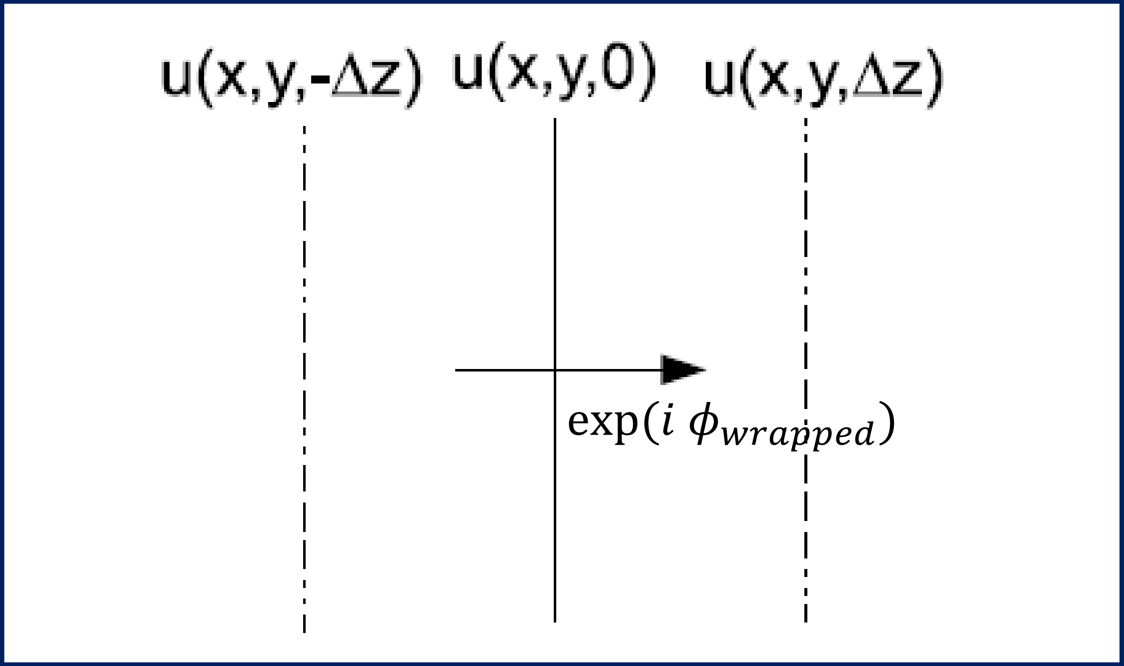


Contact profilometer vs TIE

RMS error compared to surface equation ~ $\lambda/5$

TIE offers powerful alternative to interferometry for bypassing the high fringe density problem.

Stand-alone fast phase unwrapping using TIE
[Appl. Opt. (2016)]



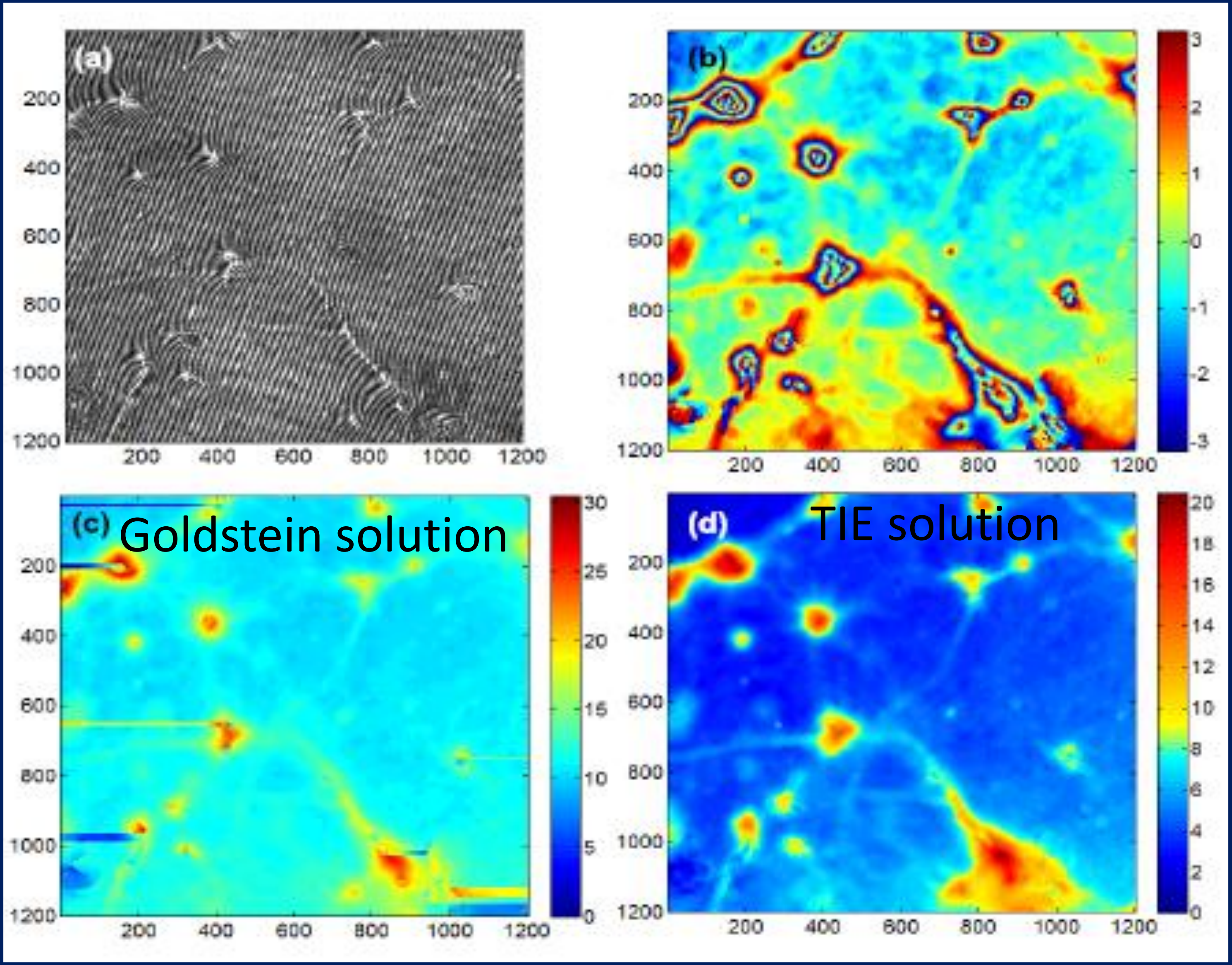
TIE

$$-k\frac{\partial I}{\partial z} = \nabla \cdot (I\, \nabla \phi)$$

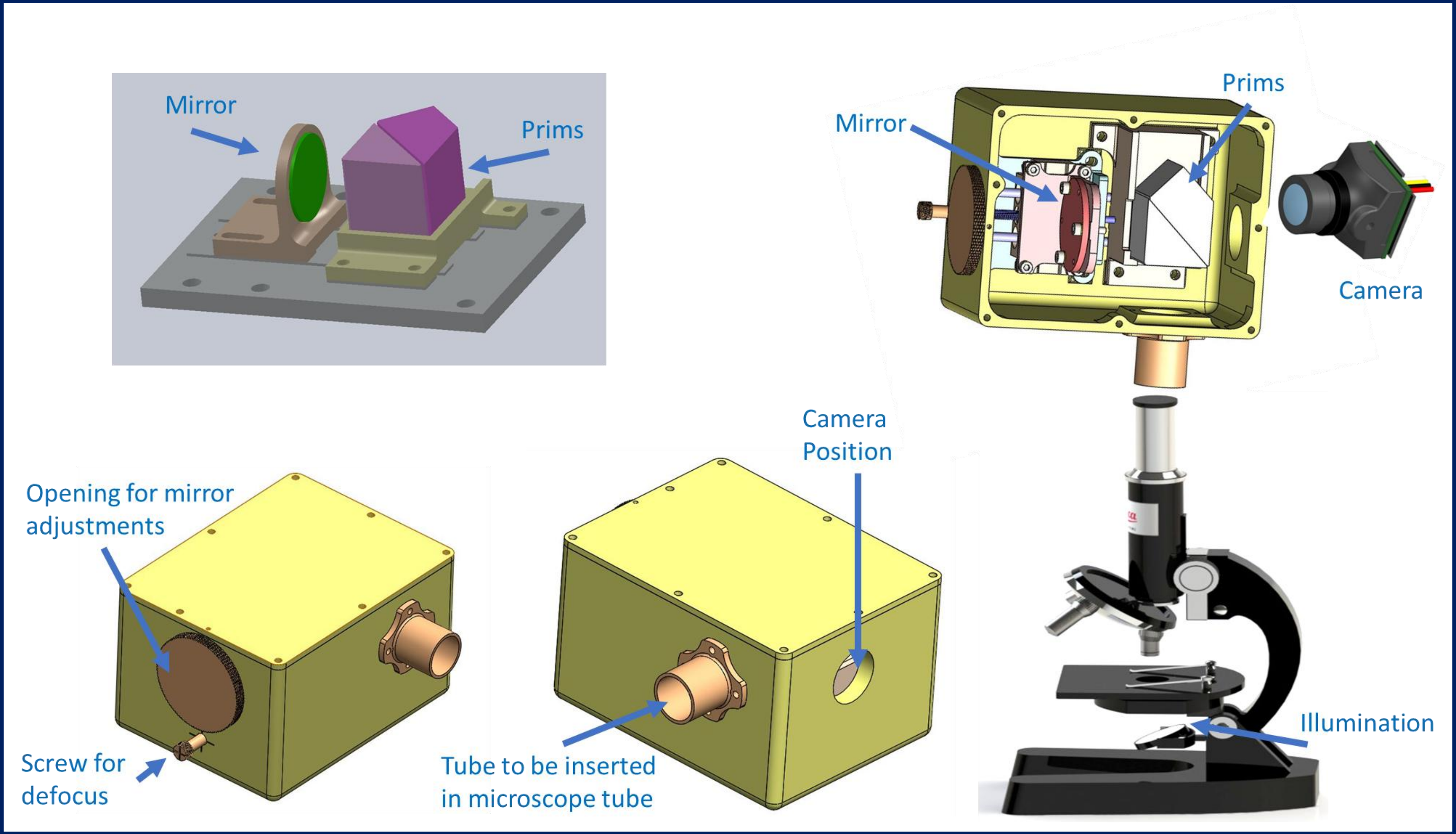
1. Create an auxiliary field using wrapped phase
2. Propagate the field by $\pm\Delta z$ (small distance for better accuracy derivative)
3. Solve TIE to get unwrapped phase

All steps implemented using FFTs ... fast algorithm

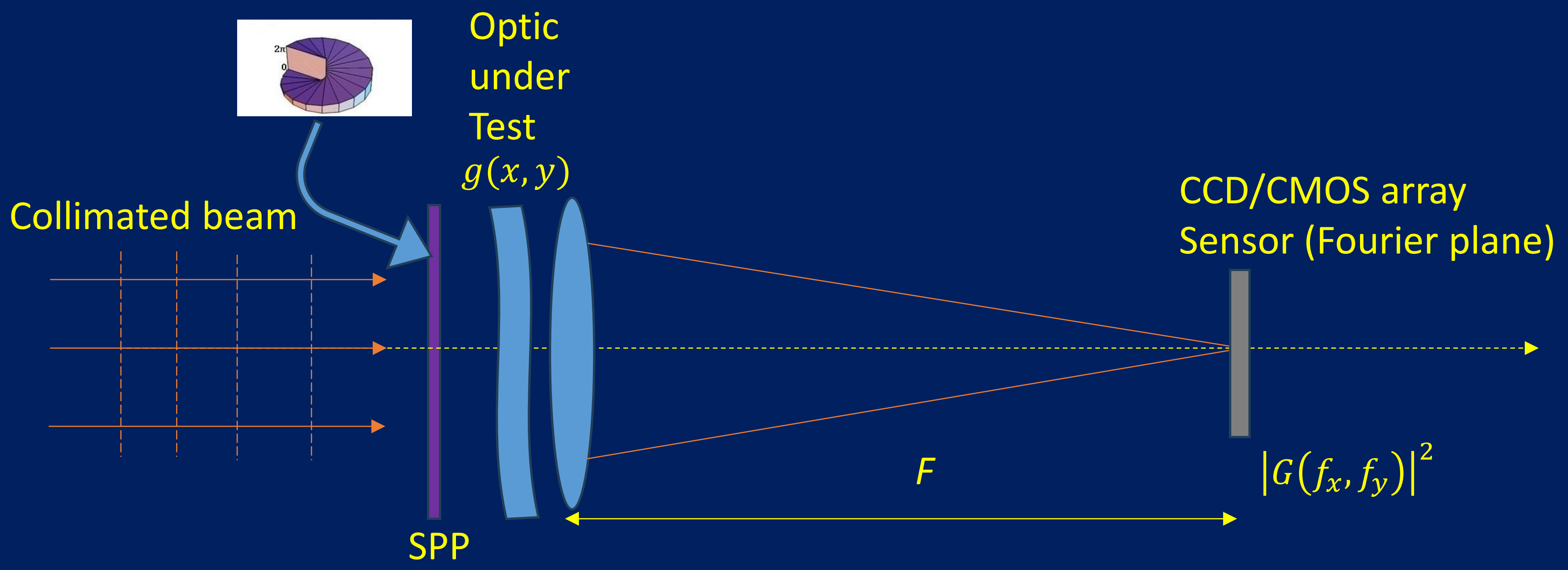
Fast unwrapping of phase using TIE [Appl. Opt. (2016)]



Compact wavefront sensor device that can be integrated with any standard imaging system …



Topic #3: Metrology with Fourier phase retrieval



- Simpler the system … harder is the computational problem
- GS, Error-reduction, HIO, RAAR, non-linear optimization … (~ 50 years of history)
- Introduction of a new “complexity constraint” [JOSA A (2019, 2021)] and charge-1 vortex illumination [JOSA A (2024)] and can make phase retrieval more stable.

Complexity constraint for phase retrieval [JOSA A (2019, 2021)]

Measure of complexity in the desired solution $g(x,y)$:

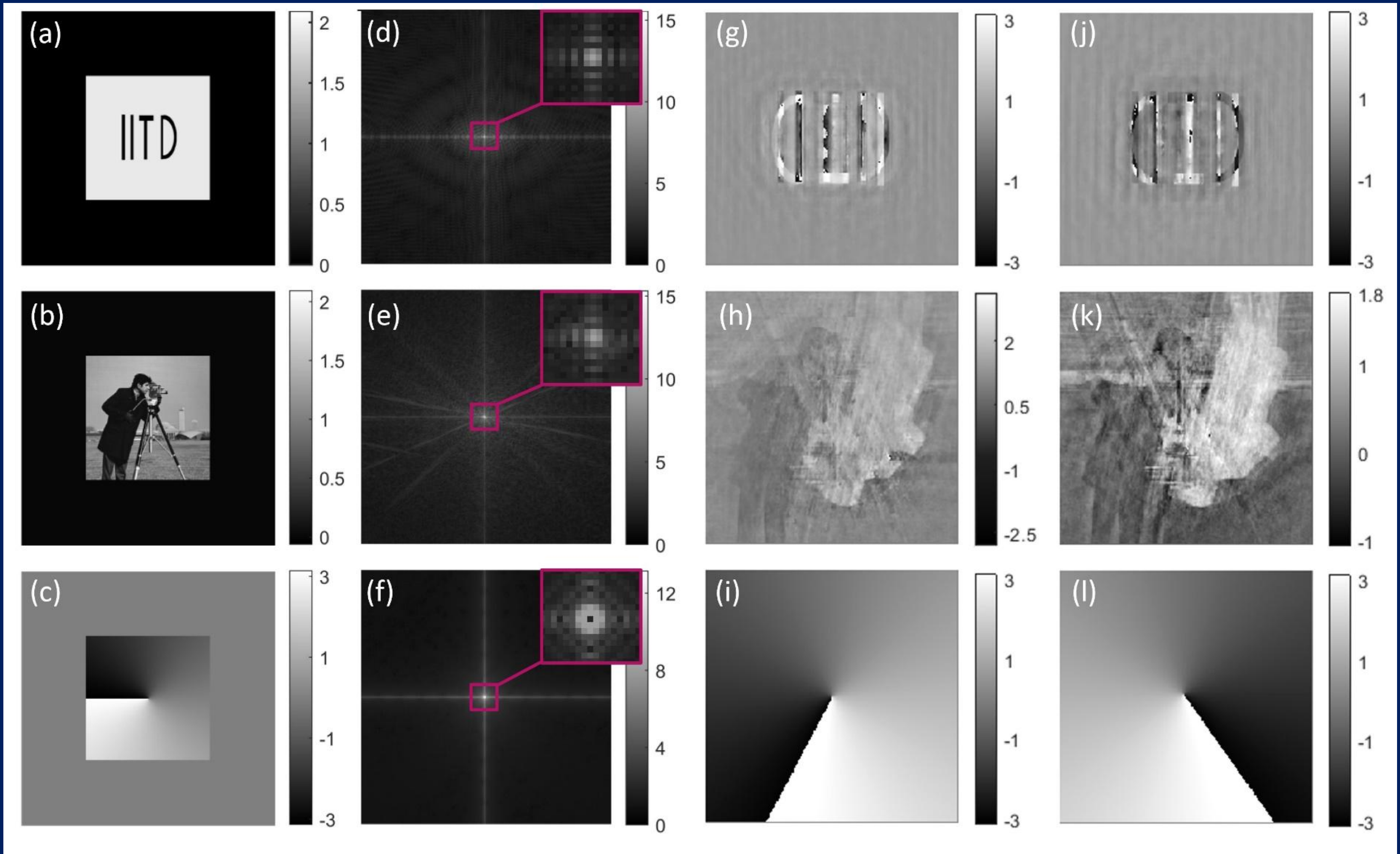
$$\zeta_o = \iint dx\, dy \;\left(|\nabla_x g|^2 + |\nabla_y g|^2 \right)$$

Parseval's theorem => Complexity can be computed directly from Fourier magnitude

$$\zeta_o = 4\pi^2 \iint df_x\, df_y \;\left(f_x^2 + f_y^2 \right) |G|^2$$

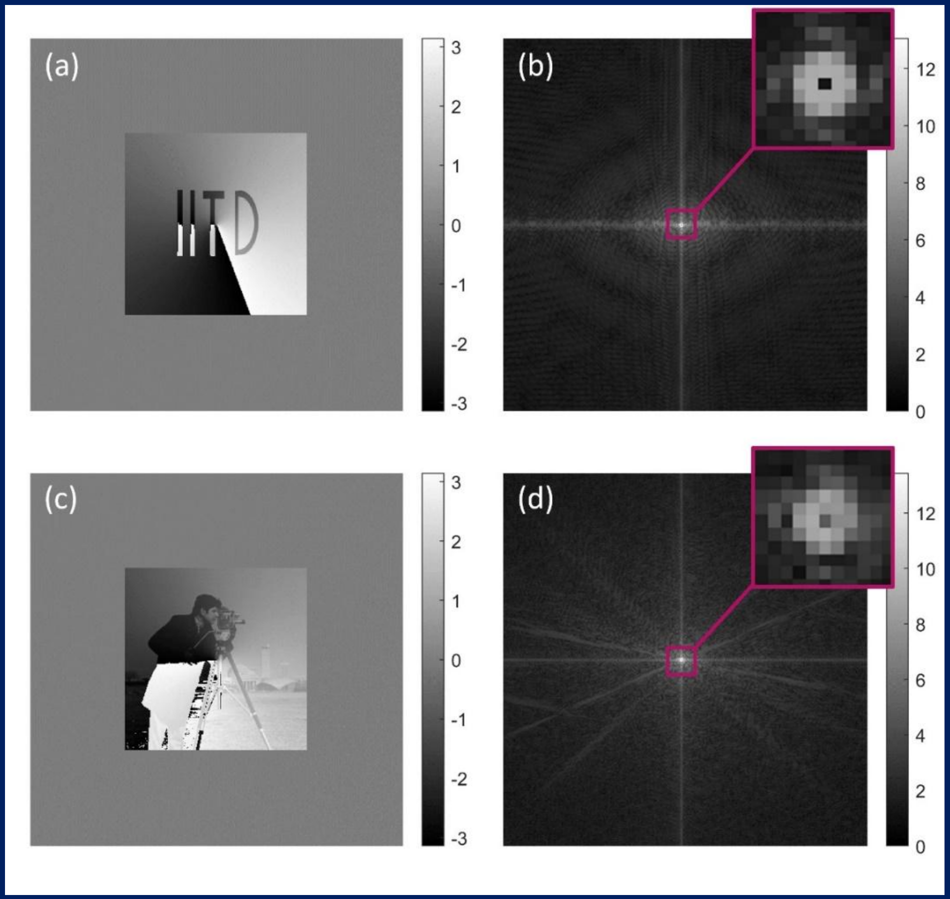
Before starting any phase retrieval algorithm, we can get an idea of fluctuations (or sparsity) in the desired solution !!

New observation regarding stagnation in phase retrieval



Typical natural phase objects show twin stagnation but a vortex phase object $\mathbf{exp}(\,i\,\theta\,)$ with $\theta = \mathbf{arctan}\left(\frac{y}{x}\right)$ does not show any twin stagnation.

Diffraction data with vortex phase illumination



- Phase map of 2D Fourier transform of limited-support functions has a vortex-like structure near isolated zeros. [Scivier & Fiddy (JOSA A 1985), Wackerman & Yagle (JOSA A 1991,1994)]

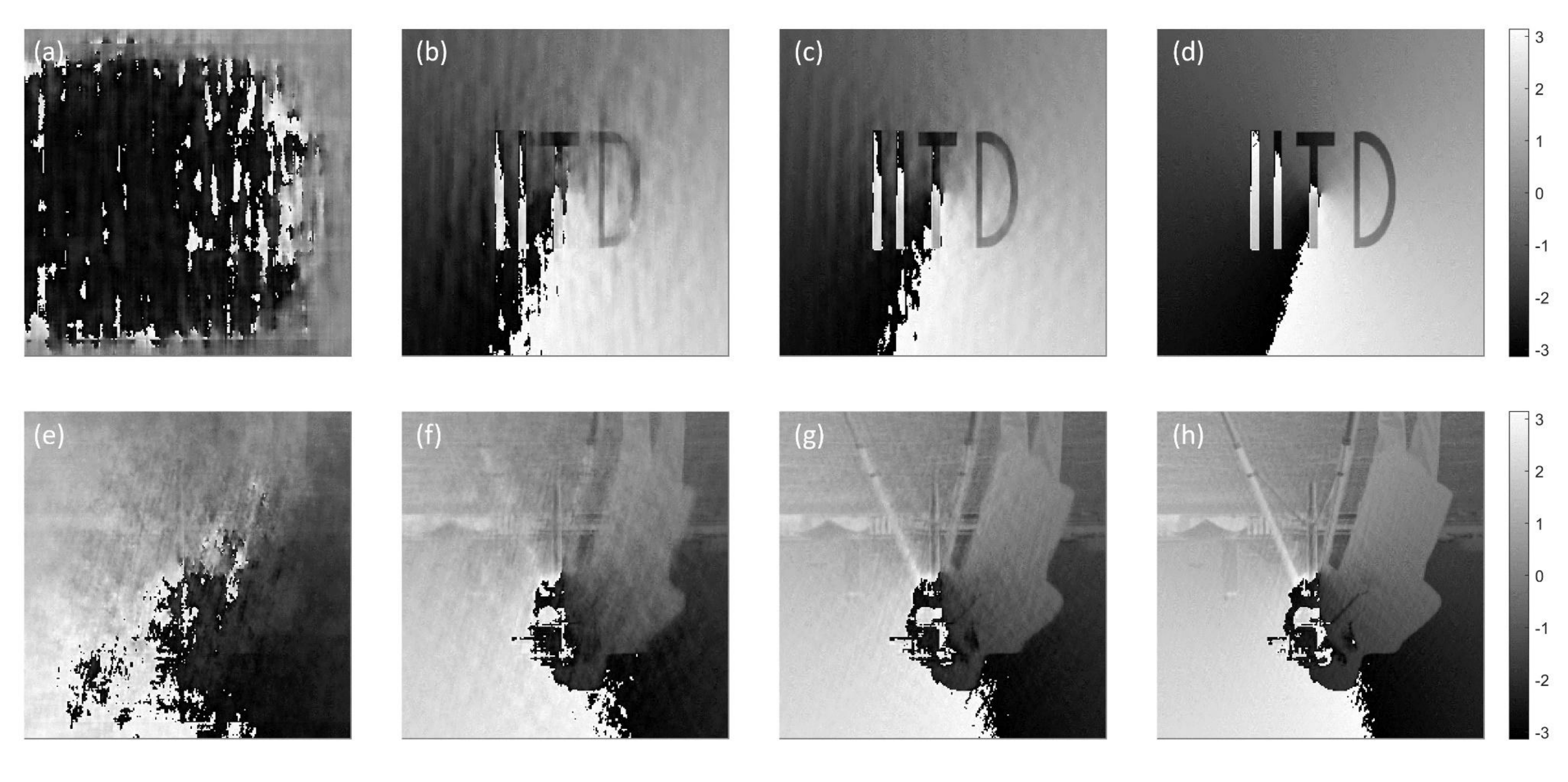
- Intentionally introduce an isolated zero in the dc spatial frequency region.

- Energy in Fourier intensity data is usually highest indc region for most natural objects.

- Phase retrieval iterations can be controlled better if a vortex of one orientation develops in this region during phase retrieval iterations.

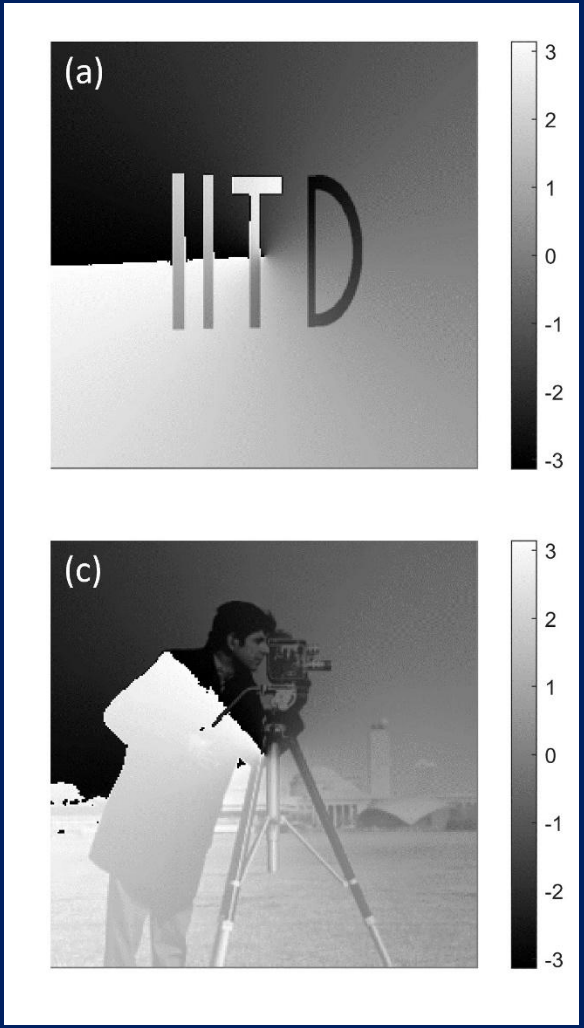
Phase solution latches onto a vortex …

One of the vortex phases develops in early iterations and the corresponding twin develops as a solution on further iterations … vortex orientation does not change later.

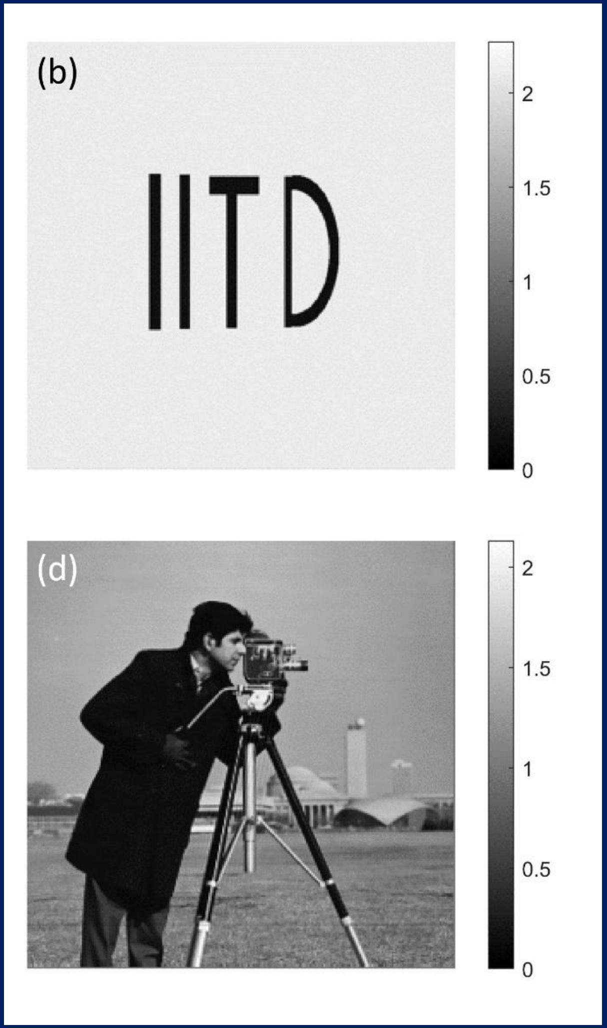


10 iterations **50 iterations** **200 iterations** **1000 iterations**

Use of (vortex illumination + complexity guidance) can stabilize iterative algorithms and provides high quality solution …



Subtract vortex phase of appropriate orientation from the solution.

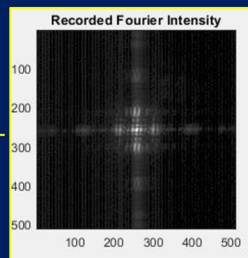


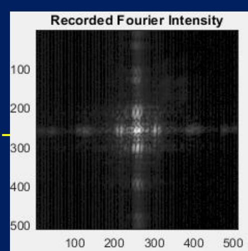
[JOSA A (2024)]

Stability of HIO iteration for plane vs vortex illumination

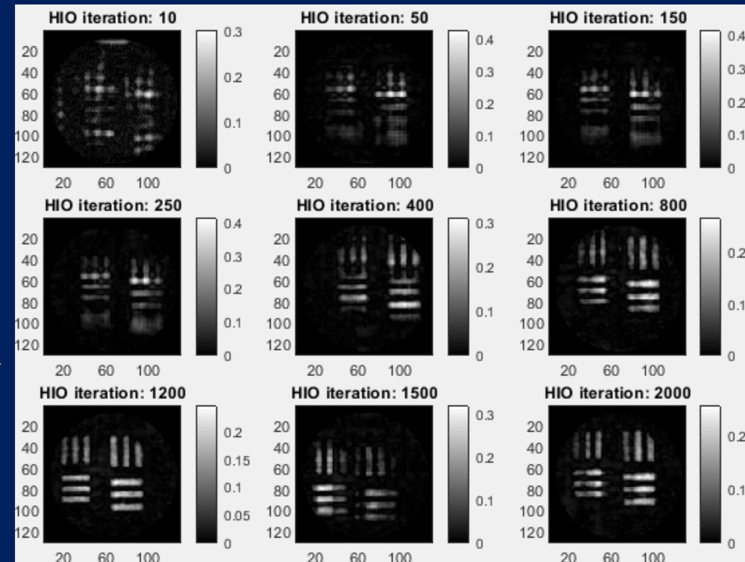
Test object

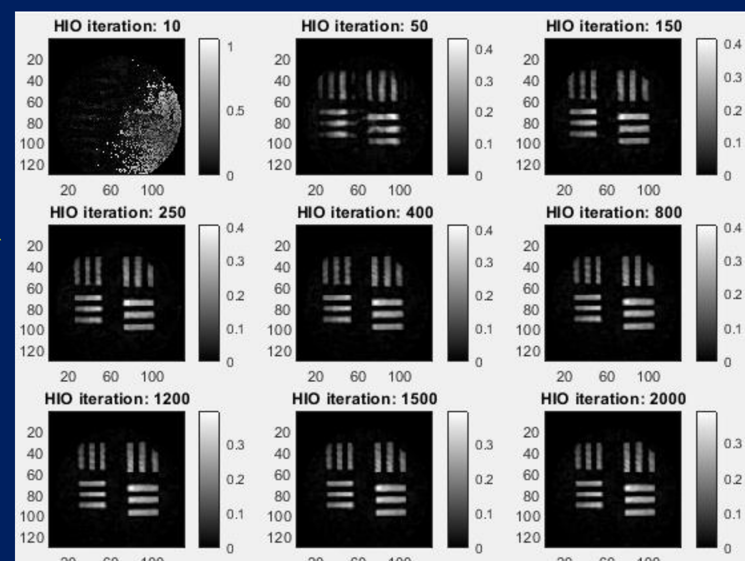
Diffraction data with plane illumination





Diffraction data with vortex illumination





Ongoing work … explore use of vortex illumination-based phase retrieval for optical metrology …

Summary:

- New algorithmic approaches offer possibility of single-shot accurate wavefront sensing and phase imaging devices

- (i) Interferometric phase recovery with optimization
  (ii) Addressing high fringe density sampling problem with TIE phase retrieval
  (iii) Metrology with vortex-illumination based coherent diffraction imaging

- Simpler hardware setups with less sensitivity to external vibrations

- Potential for portable wavefront sensors for in-situ metrology

- Dedicated efforts required to fine-tune phase reconstruction algorithms, system and detector calibration, etc. (formal optimization ideas, AI tools, … offer value in this direction)

- We are open to collaboration … drop me a message at kedark@iitd.ac.in

Thank you!

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