

## Generation and Characterization of Structured Partially Coherent Light

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## Florianópolis, SC





### GIQSUL at UFSC Group of Quantum Information



### **Optical Processing Laboratory**



### Quantum Optics Laboratory



## **GIQSUL** Research

- Transverse Spatial Entanglement with Parametric Downconversion Spatial correlations in parametric down-conversion, Walborn et al., Physics Reports 495, 87-139 (2010)
- Quantum Computing Theory (Duzzioni team)
   Quantum computation in continuous time using dynamic invariants, Sarandi et al., Physics Letters A, 375, 3343-3347 (2011)

### - Nonlinear Optics

Conservation of orbital angular momentum in stimulated downconversion, Caetano et al. Phys. Rev. A 66, 041801(R) (2002)

### - Optical Processing

An optical processor for matrix-by-vector multiplication: an application to the distance geometry problem in 1D, Hengeveld et al. Journal of Optics 24, 015701 (2021)

## <u>Outline</u>

- Introduction to optical coherence
- The Gaussian Schell Model Beam (GSM)
- The Twisted Gaussian Schell Model Beam (TGSM)
- Motivation for TGSM beams
- Generation of TGSM beams
- Quantum effects with TGSM beams
- StimPDC with TGSM beams
- Conclusions and perspectives

## Temporal and transverse spatial coherence



## Measuring transverse spatial coherence with double-slit interference



## Coherence and Double-slit interference

d

Almost monochromatic light ~ single frequency



Intensity distribution

 $k = 2\pi/\lambda$ 

$$I(p) = I_0(y)(1 + |\mu_{12}| \cos[k(d_2 - d_1) + \varphi])$$

$$x^{2} = \sqrt{x^{2} + \left(y - \frac{h}{2}\right)^{2}}; d_{2} = \sqrt{x^{2} + \left(y + \frac{h}{2}\right)^{2}}$$

 $\mu_{12}$  Normalized degree of mutual coherence

## Van Cittert-Zernike theorem



## Van Cittert-Zernike theorem

$$\mu_{12}[(x_2 - x_1), (y_2 - y_1)] = \frac{e^{i\alpha_{12}} \int_{\sigma} dx_0 dy_0 I(x_0, y_0) e^{i\frac{k}{R}[x_0(x_2 - x_1) + y_0(y_2 - y_1)]}}{\int_{\sigma} dx_0 dy_0 I(x_0, y_0)}$$



## The Gaussian Schell Model (GSM)

### THE MULTIPLE PLATE ANTENNA

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### ALLAN CARTER SCHELL

- S.B., Massachusetts Institute of Technology (1956)
- S.M., Massachusetts Institute of Technology (1956)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY September, 1961 THE MULTIPLE PLATE ANTENNA

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ALLAN CARTER SCHELL

Submitted to the Department of Electrical Engineering on August 21, 1961 in partial fulfillment of the requirements for the degree of Doctor of Science



A Sketch of a Multiple Plate Radio Astronomy Antenna

## The Gaussian Schell Model (GSM) beams

GAUSSIAN SCHELL-MODEL BEAMS

Leonard Mandel · Emil Wolf Editors



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Ari T. Friberg

Department of Technical Physics Helsinki University of Technology SF-02150 Espoo 15, Finland

> Proceedings of the Fifth Rochester Conference on Coherence and Quantum Optics held at the University of Rochester, June 13-15, 1983

## The Gaussian Schell Model (GSM) beams

In the Schell-model approximation the source cross-spectral density function takes the form

$$W(\underline{\rho_{1}}, 0; \underline{\rho_{2}}, 0) = [I(\underline{\rho_{1}}, 0)I(\underline{\rho_{2}}, 0)]^{\frac{1}{2}} \mu(\underline{\rho_{1}} - \underline{\rho_{2}}; 0)$$

GAUSSIAN SCHELL-MODEL BEAMS

Ronald J. Sudol

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 $I(\rho, 0) = A \exp\{-\rho^{2}/2\sigma_{I}^{2}\}\$  $\mu(\rho'; 0) = \exp\{-\rho'^{2}/2\sigma_{I}^{2}\}\$ 

Fig. 1. The behavior of the beam radius w(z) and the radius of curvature R(z) of a Gaussian Schell-model beam as a function of the dimensionless variable  $\xi = \lambda \pi / \pi w_0^2$  for several values of the parameter  $\alpha = \sigma_{\mu}/2\sigma_I$ . The region  $\alpha << 1$  correspond to globally incoherent beams (Gaussian quasi-homogeneous beams), whereas the limit  $\alpha \rightarrow \infty$  represents a fully coherent Gaussian laser beam.

## The Gaussian Schell Model (GSM) beams



Fig. 1. The behavior of the beam radius w(z) and the radius of curvature R(z) of a Gaussian Schell-model beam as a function of the dimensionless variable  $\xi = \lambda \pi / \pi w_0^2$  for several values of the parameter  $\alpha = \sigma_{\mu}/2\sigma_{I}$ . The region  $\alpha << 1$  correspond to globally incoherent beams (Gaussian quasi-homogeneous beams), whereas the limit  $\alpha \rightarrow \infty$  represents a fully coherent Gaussian laser beam.

R. Simon and N. Mukunda

Vol. 10, No. 1/January 1993/J. Opt. Soc. Am. A 95

## **Twisted Gaussian Schell-model beams**



### **Rajiah Simon**



### Narasimhaiengar Mukunda

R. Simon and N. Mukunda Vol.

Vol. 10, No. 1/January 1993/J. Opt. Soc. Am. A 95

**Twisted Gaussian Schell-model beams** 

We may ask, What is the most general Gaussian crossspectral density (in a transverse plane) that is invariant under arbitrary rotations about the z axis? The answer

$$\begin{split} E(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) &= a_1({\rho_1}^2 + {\rho_2}^2) + \gamma \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 \\ &+ i a_2({\rho_1}^2 - {\rho_2}^2) + i a_3 \boldsymbol{\rho}_1 \wedge \boldsymbol{\rho}_2 \\ &= (a_1 + \gamma/2)({\rho_1}^2 + {\rho_2}^2) - (\gamma/2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2 \\ &+ i a_2({\rho_1}^2 - {\rho_2}^2) + i a_3 \boldsymbol{\rho}_1 \wedge \boldsymbol{\rho}_2, \end{split}$$

$$\rho \wedge \rho' = xy' - yx'$$
$$= \rho \cdot \epsilon \rho'$$
$$\epsilon = i\sigma_2 = \begin{bmatrix} 0 & 1\\ -1 & 0 \end{bmatrix}$$

R. Simon and N. Mukunda

Vol. 10, No. 1/January 1993/J. Opt. Soc. Am. A 95

## **Twisted Gaussian Schell-model beams**

### Using optically defined parameters

$$W_{z}(\boldsymbol{\rho}_{1}, \boldsymbol{\rho}_{2}; \nu) = \frac{I(\nu)}{2\pi\sigma_{s}(\nu)^{2}} \\ \times \exp\left[\frac{-1}{4\sigma_{s}(\nu)^{2}}(\boldsymbol{\rho}_{1}^{2} + \boldsymbol{\rho}_{2}^{2}) - \frac{(\boldsymbol{\rho}_{1} - \boldsymbol{\rho}_{2})^{2}}{2\sigma_{g}(\nu)^{2}} \\ \times \frac{-i}{2\chi R(\nu)}(\boldsymbol{\rho}_{1}^{2} - \boldsymbol{\rho}_{2}^{2}) - i\frac{u(\nu)}{\chi}\boldsymbol{\rho}_{1} \cdot \boldsymbol{\epsilon}\boldsymbol{\rho}_{2}\right].$$
(2.2)

I is the intensity  $\infty_{s}$  is the beam width  $\omega_{g}$  is the coherence length

R is radius of curvature

u is the twist phase parameter

$$\chi = \lambda/2\pi$$

## **Twisted Gaussian Schell-model beams**

## Effects on divergence



## Effect on the propagation phase



# The Twisted Gaussian Schell Model (TGSM) beams illustrating beam rotation

### Article

**Statistical Characteristics of a Twisted Anisotropic Gaussian Schell-Model Beam in Turbulent Ocean** 

Yonglei Liu <sup>1,2</sup>, Yuefeng Zhao <sup>1,2</sup>, Xianlong Liu <sup>1,2</sup>, Chunhao Liang <sup>1,2</sup>, Lin Liu <sup>3</sup>, Fei Wang <sup>3</sup> and Yangjian Cai <sup>1,2,3,\*</sup>

*Photonics* **2020**, *7*, 37; doi:10.3390/photonics7020037

photonics



## **TGSM** beams motivation

Classical Optics: Robustness against Propagation in turbulent media

F. Wang and Y. Cai, "Second-order statistics of a twisted Gaussian Schell-model beam in turbulent atmosphere," *Opt. Express*, vol. 18, p. 24661, 2010.

M. Zhou, W. Fan, and G. Wu, "Evolution properties of the orbital angular momentum spectrum of twisted Gaussian Schell-model beams in turbulent atmosphere," *J. Opt. Soc. Am. A*, vol. 37, p. 142, 2020.

Y. Liu, X. Liu, L. Liu, F. Wang, Y. Zhang, and Y. Cai, "Ghost imaging with a partially coherent beam carrying twist phase in a turbulent ocean: a numerical approach," *Appl. Sci.*, vol. 9, 2019, Art no. 3023.

### Quantum Optics: Robustness against Propagation in turbulent media

Samukelisiwe Purity Phehlukwayo , Marie Louise Umuhire , Yaseera Ismail, Stuti Joshi , and Francesco Petruccione , Influence of coincidence detection of a biphoton state through free-space atmospheric turbulence using a partially spatially coherent pump, Phys. Rev. A 102, 033732 (2020)

> Quantum Optics: Boosting quantum entanglement

L. Hutter, G. Lima, and S. P. Walborn, "Boosting entanglement generation in down-conversion with incoherent illumination," *Phys. Rev. Lett.*, vol. 125, p. 193602, 2020.

### 1818 J. Opt. Soc. Am. A/Vol. 11, No. 6/June 1994

## Interpretation and experimental demonstration of twisted Gaussian Schell-model beams

Ari T. Friberg, Eero Tervonen, and Jari Turunen

Department of Technical Physics, Helsinki University of Technology, FIN- 02150 Espoo, Finland



Fig. 1. Astigmatic optical lens system used for converting an anisotropic GSM beam into a twisted GSM beam.



Fig. 5. Experimental arrangement: AOD, acousto-optic deflector;  $L_1$  and  $L_2$ , spherical lenses;  $C_1-C_6$ , cylindrical lenses; S, spatial filter.



**Fig. 1.** Experimental setup for generating a TGSM beam. DPSS, diode-pumped solid-state laser; BE, beam expander; RM, reflecting mirror; CL<sub>0</sub>, CL<sub>1</sub>, CL<sub>2</sub>, CL<sub>3</sub>, and CL<sub>4</sub>, thin cylindrical lenses;

RGGD, rotating ground glass disk;  $L_1$ ,  $L_2$ , and  $L_3$ , thin lenses; SLM, spatial light modulator; CA, circular aperture; CCD, charge-coupled device; PC<sub>1</sub> and PC<sub>2</sub>, personal computers.

RM



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Nanophotonics 2022; 11(4): 689-696

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### **Research Article**

Haiyun Wang, Xiaofeng Peng, Hao Zhang, Lin Liu\*, Yahong Chen\*, Fei Wang\* and Yangjian Cai\*

## Experimental synthesis of partially coherent beam with controllable twist phase and measuring its orbital angular momentum



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Nanophotonics 2022; 11(4): 689-696

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### **Research Article**

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# Experimental synthesis of partially coherent beam with controllable twist phase and measuring its orbital angular momentum



## SLM movie method for generating TGSM beams



## Parametric Down-conversion



## Spatial Entanglement Can Be Measured And Witnessed



## Spatial Entanglement Can Be Witnessed

Lu-Ming Duan, G. Giedke, J. I. Cirac, and P. Zoller Phys. Rev. Lett. 84, 2722 (2000).

### DGCZ criterion



### MGVT criterion

S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi Phys. Rev. Lett. 88, 120401 (2002).



## Spatial Entanglement Can Be Measured And Witnessed



Incoherence degrades spatial correlations

## PHYSICAL REVIEW A 99, 053831 (2019)

Spatially entangled photon-pair generation using a partial spatially coherent pump beam

Hugo Defienne\* and Sylvain Gigan





## Pumping SPDC with partially coherent pump

## PHYSICAL REVIEW A 102, 033732 (2020)

Influence of coincidence detection of a biphoton state through free-space atmospheric turbulence using a partially spatially coherent pump

Samukelisiwe Purity Phehlukwayo<sup>1</sup>, Marie Louise Umuhire<sup>1</sup>, Yaseera Ismail,<sup>1,\*</sup> Stuti Joshi<sup>2,†</sup> and Francesco Petruccione<sup>1,3,‡</sup>





## Incoherence degrades spatial correlations

**Research Article** 

Vol. 27, No. 15 | 22 Jul 2019 | OPTICS EXPRESS 20745

### **Optics EXPRESS**

Influence of pump coherence on the generation of position-momentum entanglement in optical parametric down-conversion

WUHONG ZHANG,<sup>1,2</sup> ROBERT FICKLER,<sup>2,3</sup> ENNO GIESE,<sup>2,4,6</sup> LIXIANG CHEN,<sup>1,7</sup> AND ROBERT W. BOYD<sup>2,5</sup>





## TGSM beam pumping can increase entanglement!

### PHYSICAL REVIEW LETTERS 125, 193602 (2020)

**Boosting Entanglement Generation in Down-Conversion with Incoherent Illumination** 

Lucas Hutter<sup>1,2</sup> G. Lima,<sup>3,4</sup> and S. P. Walborn<sup>1,3,4</sup>

A TGSM beam is therefore uniquely characterized by its CM [23,24]:

$$T = \begin{pmatrix} \sigma^2 & -\frac{k\sigma^2}{R} & 0 & ku\sigma^2 \\ -\frac{k\sigma^2}{R} & \tau^2 & -ku\sigma^2 & 0 \\ 0 & -ku\sigma^2 & \sigma^2 & -\frac{k\sigma^2}{R} \\ ku\sigma^2 & 0 & -\frac{k\sigma^2}{R} & \tau^2 \end{pmatrix}.$$
 (1)

 $\sigma$  is the beam waist

 $\tau^2 = (1/\delta^2) + (1/4\sigma^2) + k^2[(\sigma^2/R^2) + u^2\sigma^2)]$ 

is the variance of the wave vector distribution.

 $\delta$  is the transverse coherence length,





## TGSM beam pumping can increase entanglement!

Two-photon covariance matrix

 $V_{12} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \tag{4}$ 

A (B) refer to photon 1 (2) A = B

$$A = \begin{pmatrix} \sigma^2 + \sigma_-^2 & -\frac{k\sigma^2}{2R} & 0 & \frac{ku\sigma^2}{2} \\ -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 + \Delta_-^2) & -\frac{ku\sigma^2}{2} & 0 \\ 0 & -\frac{ku\sigma^2}{2} & \sigma^2 + \sigma_-^2 & -\frac{k\sigma^2}{2R} \\ \frac{ku\sigma^2}{2} & 0 & -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 + \Delta_-^2) \end{pmatrix}$$

L is the length of the crystal

$$\sigma_{-}^2 = 9L/10k$$

 $\Delta^2 = 3k/2L$ 

$$C = \begin{pmatrix} \sigma^2 - \sigma_-^2 & -\frac{k\sigma^2}{2R} & 0 & \frac{ku\sigma^2}{2} \\ -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 - \Delta_-^2) & -\frac{ku\sigma^2}{2} & 0 \\ 0 & -\frac{ku\sigma^2}{2} & \sigma^2 - \sigma_-^2 & -\frac{k\sigma^2}{2R} \\ \frac{ku\sigma^2}{2} & 0 & -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 - \Delta_-^2) \end{pmatrix}$$

The symplectic eigenvalues of Eq. (4) are twofold degenerate and given by [37]

$$\lambda_{\pm} = \frac{1}{\sqrt{2}} \left| \sqrt{a_{\pm} \pm \sqrt{4k^2 \Delta_{-}^2 \sigma_{-}^2 \sigma_{-}^4 \left(u^2 \pm \frac{1}{R^2}\right) + a_{-}^2}} \right|, \quad (8)$$

Entanglement is confirmed when  $\lambda_{-} < 1/2$  (gray horizontal plane). The SPDC parameters are  $R = \infty$ ,  $\lambda_p = 400$  nm,  $\sigma_p = 50 \ \mu$ m, and L = 1 cm. (b) Profile plots of  $\lambda_{-}$  for normalized twist phase  $|u|/k\delta^2$  equal to zero (black solid line), 1 (red solid line), and 1/2 (blue dashed line). The dotted black curve is the near-field and far-field entanglement criteria (7).



### **Research Article**

Gustavo H. dos Santos, Andre G. de Oliveira, Nara Rubiano da Silva, Gustavo Cañas, Esteban S. Gómez, Stuti Joshi, Yaseera Ismail, Paulo H. Souto Ribeiro and Stephen Patrick Walborn\*

# Phase conjugation of twisted Gaussian Schell model beams in stimulated down-conversion



## Partially coherent StimPDC

## StimPDC with twisted Schell-model beams

$$W_i(\mathbf{r},\mathbf{r}') = W_p(\mathbf{r},\mathbf{r}')W_s^*(\mathbf{r},\mathbf{r}').$$



$$W_{i}(\mathbf{r},\mathbf{r}') = A e^{-\frac{r^{2}+r'^{2}}{4w_{i}^{2}}} e^{-\frac{(\mathbf{r}-\mathbf{r}')^{2}}{2\delta_{i}^{2}}} e^{-ik_{i}\frac{(\mathbf{r}-\mathbf{r}')^{2}}{2R_{i}}} e^{-ik_{i}\mu_{i}(xy'-yx')},$$



$$\frac{k_i}{R_i} = \frac{k_p}{R_p} - \frac{k_s}{R_s},$$

$$k_i\mu_i=k_p\mu_p-k_s\mu_s,$$

 $\delta_s^2 \delta_p^2$ 

## Experiment

Pump @ 405 nm Seed @ 780 nm Idler @ 840 nm



## Results

## Far field variance x degrade of coherence

## Far field variance

$$\sigma_{\rm ff}^2 = \left(\frac{1}{4w^2} + \frac{k^2w^2}{R^2}\right) + \frac{1}{\delta^2} + \frac{\tau^2w^2}{\delta^4},$$



## Visibility in a double-slit interference



## Visibility x degree of coherence



### Transfer and conjugation of twist phase

$$I(\mathbf{r}) \approx \left[1 + \mathrm{e}^{-\frac{2d^2}{\delta^2}} \cos\left\{2dk\left(\frac{y}{f} - \mu x\right)\right\}\right]$$

## Twist in the fringe pattern



Journal of Optics

J. Opt. 24 (2022) 094004 (8pp)

## Evaluation of twisted Gaussian Schell model beams produced with phase randomized coherent fields

G Cañas<sup>1,2</sup><sup>(i)</sup>, E S Gómez<sup>2,3</sup>, G H dos Santos<sup>4</sup><sup>(i)</sup>, A G de Oliveira<sup>4</sup>, N Rubiano da Silva<sup>4</sup>, Stuti Joshi<sup>5</sup>, Yaseera Ismail<sup>6</sup>, P H S Ribeiro<sup>4,\*</sup><sup>(i)</sup> and S P Walborn<sup>2,3,\*</sup>



## Far field mean variance versus coherence length



### IOP Publishing

Journal of Optics

### J. Opt. 24 (2022) 094004 (8pp)

#### https://doi.org/10.1088/2040-8986/ac8562



# Residual coherence in SLM movie methods







PHYSICAL REVIEW APPLIED 20, 024007 (2023)

### Partial Coherence and Coherence Length in Stimulated Parametric Down-Conversion

G.H. dos Santos<sup>(0)</sup>,<sup>1,\*</sup> R.C. Souza Pimenta<sup>(0)</sup>,<sup>1</sup> R.M. Gomes,<sup>2</sup> S.P. Walborn,<sup>3,4,†</sup> and P.H. Souto Ribeiro<sup>1,‡</sup>



TABLE I. Normalized intensity of the stimulated component and  $\epsilon$ -coherence lengths obtained for different seed beam intensities.

Iseed (µW)	β (%)	$\epsilon = 1/e$ (µm)	$\epsilon = 1/2$ (µm)	$\epsilon = 7/8$ (µm)
0	0	29.30	24.40	10.71
90	$4.9 \pm 3.5$	30.64	25.32	11.00
270	$13.9 \pm 2.7$	33.74	27.32	11.61
540	$20.5 \pm 2.5$	36.88	29.17	12.12
3000	$44.9 \pm 1.8$	$\infty$	45.30	14.87
10 700	$76.0\pm1.7$	$\infty$	$\infty$	25.12





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### Optics and Laser Technology

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Optics & Laser Technology

Full length article

Anomalous second harmonic generation of twisted Gaussian Schell model beams

M. Gil de Oliveira <sup>a,\*</sup>, A.L.S. Santos Junior <sup>a</sup>, A.C. Barbosa <sup>a</sup>, B. Pinheiro da Silva <sup>a</sup>, G.H. dos Santos <sup>b</sup>, G. Cañas <sup>c,d</sup>, P.H. Souto Ribeiro <sup>b</sup>, S.P. Walborn <sup>c,e</sup>, A.Z. Khoury <sup>a</sup>







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## Conclusions

Partial coherence GSM and TGSM beams

TGSM beams: applications to communication through turbulent medium

TGSM and SPDC applications to quantum systems TGSM and wavelength conversion in StimPDC

StimPDC as a design for SPDC experiments in quantum regime

Perspectives

Testing twist conservation in nonlinear parametric interactions

Testing experimentally the use of TGSM to boost the entanglement

Using TGSM beams in optical communication REDE RIO QUÂNTICA (A. Z. Khoury)



## Whenk you!



	Infrared			Green		
$\tau = -1$	$\tau = 0$	$\tau = 1$	$\tau = -1$	$\tau = 0$	$\tau = 1$	1.0
						0.8
9						
						sity (a.
•••			-		-	0.4 <sup>m</sup>
						-0.2
<b>~</b>						1.0.0
	τ = -1	$\tau = -1$ $\tau = 0$	Infrared $\tau = -1$ $\tau = 0$ $\tau = 0$ $\tau = 1$ $\bullet$	$\begin{array}{c c} \text{Infrared} \\ \tau = -1 & \tau = 0 & \tau = 1 \\ \hline \bullet & \bullet & \bullet & \bullet \\ \hline $	Infrared Green $\tau = -1$ $\tau = 0$ $\tau = 1$ $\tau = -1$ $\tau = 0$ $\tau = -1$ $\tau = 0$	Infrared $\tau = -1$ $\tau = 0$ $\tau = 1$ $\tau = -1$ $\tau = 0$ $\tau = 1$ $\tau = -1$ $\tau = 0$ $\tau = 1$ $\tau $





